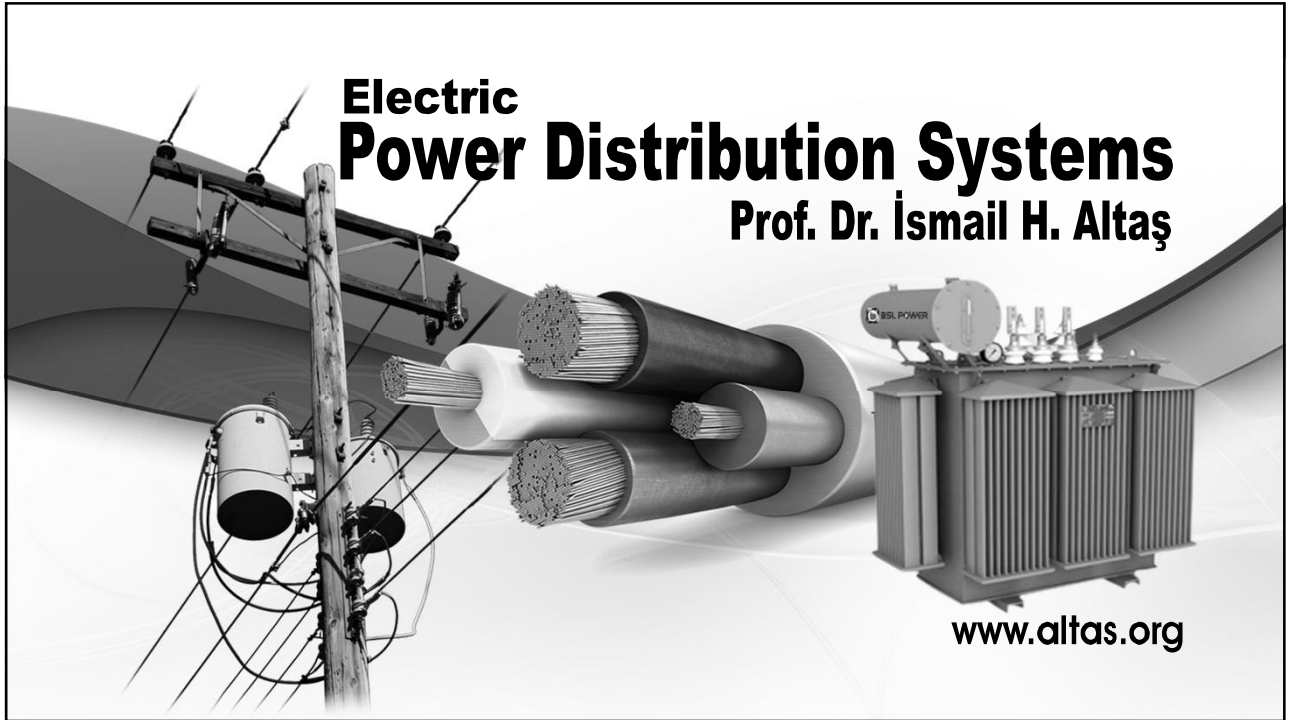


# Electric Power Distribution Systems

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## Electric Power Distribution Systems

### CHAPTER 6

### CAPACITORS IN DISTRIBUTION SYSTEMS

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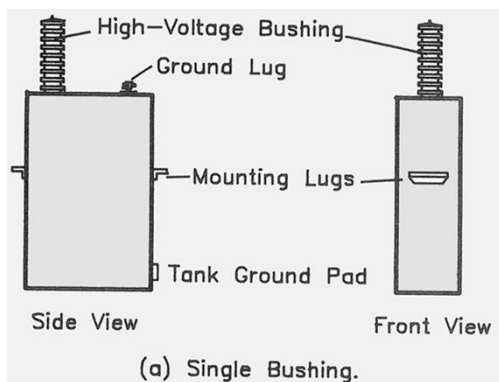
# Introduction

The application of capacitors to electrical power systems can produce several desirable effects. Improved voltage regulation, power factor correction, reduced line losses, and released system capacity are a few of the advantages. Capacitors are usually installed on a power system in a three-phase configuration, rather than single phase. The individual capacitor units making up a bank may be either three phase or single phase. Capacitors may be installed on the customer service or on the utility system.

The primary function of capacitors is to supply reactive power to the system. As a result, capacitors supply a portion of the reactive power required by various lagging power factor loads in the system. A reduction in line current magnitude, overall system apparent power loading, and  $I^2R$  line losses are obtained. In addition, voltage drop is reduced due to the decrease in line current magnitude and improvement in power factor.

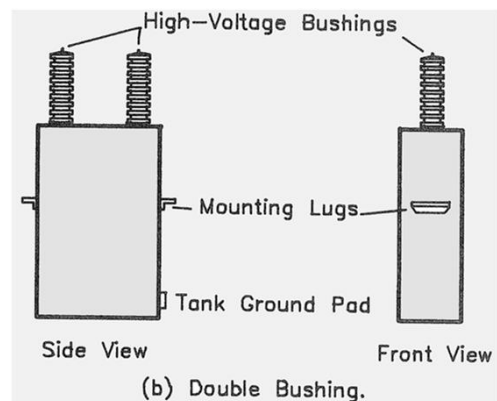
This chapter will discuss the general construction and ratings of capacitors, three-phase connections of single-phase units, switching, and control of capacitor banks. Additional emphasis will be placed on sizing capacitors for power factor improvement and locating capacitors for maximum loss reduction and voltage improvement.

## Capacitor Construction and Ratings



There is only one HV bushing for connection to the phase conductor.

Suitable for connection in a ground-wye configuration.



Two bushing, single phase, medium voltage capacitor unit.

Typically connected in DELTA or floating-wye configuration

## Capacitor Construction and Ratings

Low-voltage (less than 1000 V) capacitor units may be either single phase or three phase. The low-voltage, single-phase units typically have two bushings for connection to the line. Three-phase low-voltage units are typically supplied with three bushings for

connection to all three-phase conductors on a three-phase system. Low-voltage capacitors are usually connected in a delta configuration.

**Table 9-1** Voltage and kVAR Ratings, Single-Phase, Medium-Voltage Capacitors

<i>Voltage</i>		<i>kVAR</i>
2400	12,470	50
2770	13,280	100
4160	13,800	150
4800	14,400	200
6640	15,125	300
7200	19,920	400
7620	20,800	500
7960	21,600	
8320	22,130	
9540	22,800	
9960	23,800	
11,400	24,940	

Courtesy: Cooper Power Systems

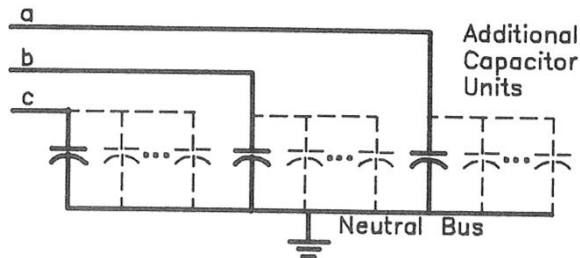
## Capacitor Construction and Ratings

In accordance with the National Electrical Code®, all capacitor units are supplied with an internal discharge resistor. The discharge resistor is connected in parallel with the capacitor unit and provides a path for current to flow in the event that the capacitor is disconnected from the source. For low-voltage capacitors, the residual voltage trapped on the capacitor unit must decrease to less than 50 V within 1 min after de-energizing. For medium-voltage capacitors, the residual voltage must decrease to less than 50 V within 5 min after de-energizing.

Table 9-1 lists standard voltage and kVAR ratings for medium-voltage capacitors. Standard voltage ratings for low-voltage capacitor units are 240, 480, and 600 V. Standard kVAR ratings for low-voltage capacitor units are too numerous to list here. The reader is referred to manufacturers' product literature.

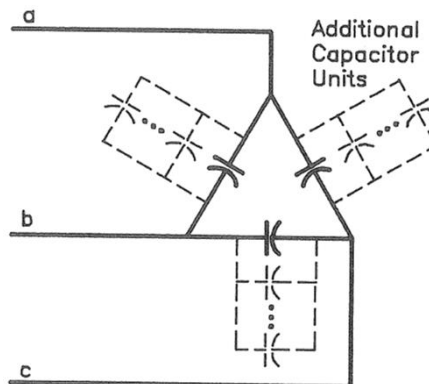
## CONNECTIONS

Capacitors installed on medium-voltage, multigrounded, neutral utility distribution systems are usually either solidly grounded wye or floating wye connected. Figure 9-2 shows some of the more common capacitor connections. A typical installation of a fixed capacitor bank on a 12.47-kV distribution feeder is shown in Fig. 9-3. A switched capacitor bank installed on a 12.47-kV distribution feeder is shown in Fig. 9-4. These capacitors are switched off and on by the oil switches located next to the capacitor units. In both figures, the capacitors are installed in a grounded-wye configuration.



(a) Solidly Grounded Wye.

## CONNECTIONS

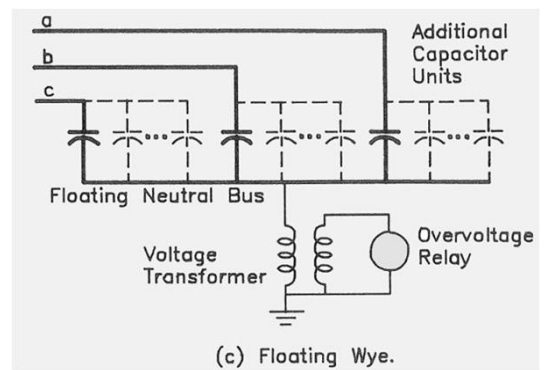
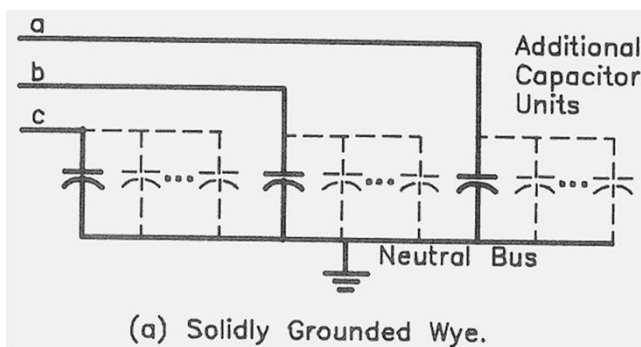


(b) Delta.



The solidly grounded wye connection shown in Fig. 9-2a is typically used on medium-voltage distribution feeders. The voltage rating of the capacitor units in the solidly grounded wye bank must be equal to or greater than the nominal line to neutral voltage of the feeder. The kVAR rating of the capacitor units is selected to provide the desired amount of reactive compensation. Additional single-phase capacitor units may be connected in parallel per phase to increase the rating of the bank. The individual capacitor units in the bank must have the same kVAR and voltage

ratings. Group fusing of the capacitors is typically provided by fused cutouts. Since the capacitor units are solidly connected between line and neutral, the failure of any individual capacitor unit will not result in an overvoltage across the remaining units in the bank.



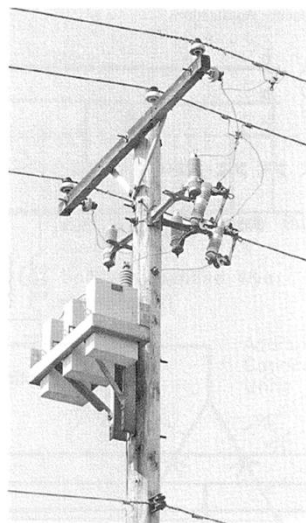
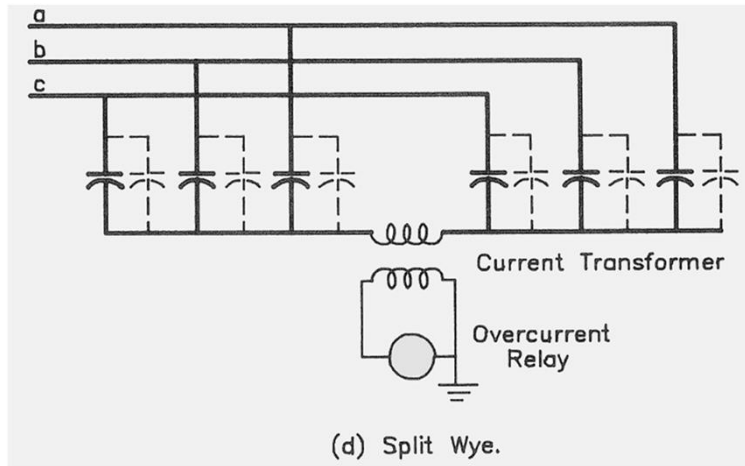
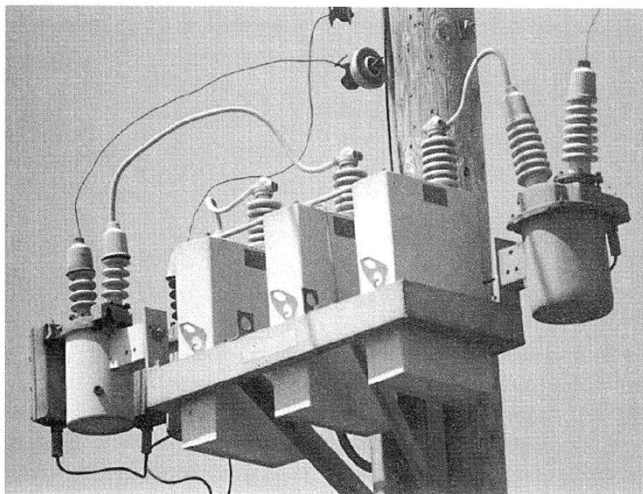


Figure 9-3 Fixed capacitor bank installation on 12.47-kV distribution feeder. (Photo Courtesy of Ohio Edison Company)



**Figure 9-4** Close-up of switched capacitor bank installation on 12.47-kV distribution feeder. (Photo Courtesy of Ohio Edison Company)

EXAMPLE

Determine the appropriate voltage and kVAR ratings for the capacitor units used to make up a 1800-kVAR, grounded-wye connected capacitor bank to be installed on a 12.47-kV, three-phase, MGN distribution feeder.

*Solution* The kVAR per phase is equal to

$$\text{kVAR/phase} = 1800 \text{ kVAR} / 3 \text{ phases} = 600 \text{ kVAR/phase}$$

Referring to Table 9-1, the following possible combinations exist:

- 12–50 kVAR units per phase
- 6–100 kVAR units per phase
- 4–150 kVAR units per phase
- 3–200 kVAR units per phase
- 2–300 kVAR units per phase

**Capacitors** Voltage-kVAR ratings  
Single-phase, Medium Voltage

Voltage	kVAR	
2400	12,470	50
2770	13,280	100
4160	13,800	150
4800	14,400	200
6640	15,125	300
7200	19,920	400
7620	20,800	500
7960	21,600	
8320	22,130	
9540	22,800	
9960	23,800	
11,400	24,940	

MGN: Multi Grounded Neutral

The most practical combination would be two 300-kVAR units per phase, for a total of six units comprising the bank. Therefore, six 300-kVAR, 7200-V capacitor units are needed for this bank. Single- or double-bushing units may be used.

The voltage rating of each capacitor unit is equal to the nominal line to neutral voltage of the system.

$$V_{\text{ln}} = \frac{12,470}{\sqrt{3}} = 7200 \text{ V}$$

In larger capacitor banks installed on distribution or subtransmission substation busses, individual fusing of the capacitor units may be employed. Figure 9-5 shows a capacitor bank installed on a 23-kV substation bus. With the solidly grounded wye connection using individual fusing of the capacitor units, a blown fuse detection scheme is needed to detect operation of any of the individual fuse elements. One such blown fuse detection scheme employs a current transformer connected between the neutral point of the bank and ground. Under normal operating conditions with all capacitor units energized, the current flow from the bank neutral to ground is zero. However, if any of the individual capacitor fuse elements operate, an unbalanced current will result. The secondary of the current transformer could be connected to an overcurrent relay to detect the unbalanced current. Tripping of the capacitor bank breaker or switch could be initiated or an alarm condition reported.

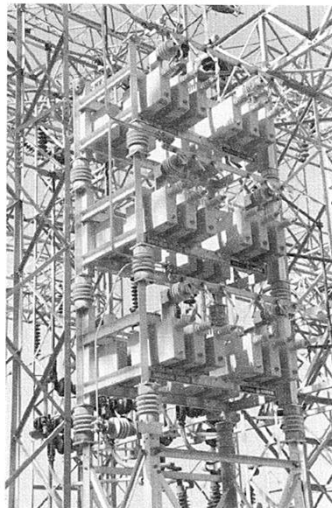


Figure 9-5 Capacitor bank installation on 23-kV substation bus. (Photo Courtesy of Ohio Edison Company)

EXAMPLE

A 4800-kVAR, 12.47-kV, solidly grounded wye capacitor bank is made up of eight 200-kVAR, 7200-V capacitor units per phase. A blown fuse detection scheme is to be used to determine the presence of a blown fuse. Assume that one fuse in phase A is blown and calculate the current flowing from the neutral of the bank to ground.

*Solution* All eight capacitor units are energized in phases B and C. The impedances in these two phases are

$$Z_B = Z_C = -j \frac{7200^2}{8 \cdot 200,000} = -j32.4 \Omega$$

The impedance in phase A is equal to

$$Z_A = -j \frac{7200^2}{7 \cdot 200,000} = -j37.0 \Omega$$

The source voltage references are selected as

$$V_{AN} = 7200 \angle 0^\circ \text{ V,}$$

$$V_{BN} = 7200 \angle -120^\circ \text{ V,}$$

$$V_{CN} = 7200 \angle 120^\circ \text{ V}$$

The resulting capacitor bank line currents are

$$I_A = \frac{7200\angle 0^\circ}{37.0\angle -90^\circ} = 194.6\angle 90^\circ \text{ A}$$

$$I_B = \frac{7200\angle -120^\circ}{32.4\angle -90^\circ} = 222.2\angle -30^\circ \text{ A}$$

$$I_C = \frac{7200\angle 120^\circ}{32.4\angle -90^\circ} = 222.2\angle 210^\circ \text{ A}$$

The neutral current is equal to

$$\begin{aligned} I_N &= -(I_A + I_B + I_C) \\ &= -(194.6\angle 90^\circ + 222.2\angle -30^\circ + 222.2\angle 210^\circ) \\ &= 27.6\angle 90^\circ \text{ A} \end{aligned}$$

Therefore, with one capacitor unit in the bank de-energized, the neutral current is equal to 27.6 A. A current transformer and overcurrent relay could be used to sense this current in the event of a fuse operation.

In the delta connection shown in Fig. 9-2b, the individual capacitor units are connected phase to phase. Therefore, the required voltage rating of the capacitor units must be equal to or greater than the nominal line to line voltage of the system. Similarly to the grounded-wye connection, the failure of any of the individual capacitor units will not result in an overvoltage across the remaining units in the bank.

EXAMPLE

Determine the appropriate voltage and kVAR ratings for the capacitor units used to make up a 2400-kVAR, delta-connected capacitor bank to be installed on a 13.8-kV, three-phase, three-wire feeder.

*Solution* The kVAR per phase is equal to

$$\text{kVAR/phase} = 2400 \text{ kVAR}/3 \text{ phases} = 800 \text{ kVAR/phase}$$

The most practical combination would be two 400-kVAR units per phase, for a total of six units comprising the bank. Therefore, six 400-kVAR, 13,800-V capacitor units are needed for this bank. Double-bushing units are required for the delta connection.

The voltage rating of each capacitor unit is equal to the nominal line to line voltage of the system:

$$V_{ll} = 13,800 \text{ V}$$

## CONNECTIONS

### DELTA CONNECTION

#### EXAMPLE

*Solution* The kVAR per phase is equal to

$$\text{kVAR/phase} = 2400 \text{ kVAR}/3 \text{ phases} = 800 \text{ kVAR/phase}$$

$$V_{ll} = 13,800 \text{ V}$$

The most practical combination would be two 400-kVAR units per phase, for a total of six units comprising the bank. Therefore, six 400-kVAR, 13,800-V capacitor units are needed for this bank. Double-bushing units are required for the delta connection.

## CONNECTIONS

### FLOATING-WYE CONNECTION

The floating-wye connection shown in Fig. 9-2c is commonly used for larger kVAR rated capacitor banks installed on distribution substation and subtransmission substation buses. In this connection, the common or neutral point of the capacitor bank is not directly connected to ground. Since the potential of the bank neutral is allowed to float

with respect to ground, the capacitor units should have two bushings. In some installations, however, single-bushing units are used, with the neutral connection made between the individual capacitor units and the capacitor mounting rack. The capacitor mounting rack itself is insulated from ground potential and is allowed to float with respect to ground. With this type of connection, it is possible for the capacitor housings to become energized.

The voltage rating of the capacitor units must be equal to or greater than the nominal line to neutral voltage of the system. Individual capacitor units may be series connected to achieve higher voltage ratings and parallel connected to achieve higher kVAR ratings. Each unit in the bank should have the same kVAR and voltage rating.

# CONNECTIONS

## FLOATING-WYE CONNECTION

### EXAMPLE

Determine the appropriate voltage and kVAR ratings for the capacitor units used to make up a 8400-kVAR, floating-wye connected capacitor bank to be installed on a 69-kV, three-phase, subtransmission substation bus.

*Solution* The kVAR per phase is equal to  
 $kVAR/phase = 8400 \text{ kVAR}/3 \text{ phases} = 2800 \text{ kVAR/phase}$

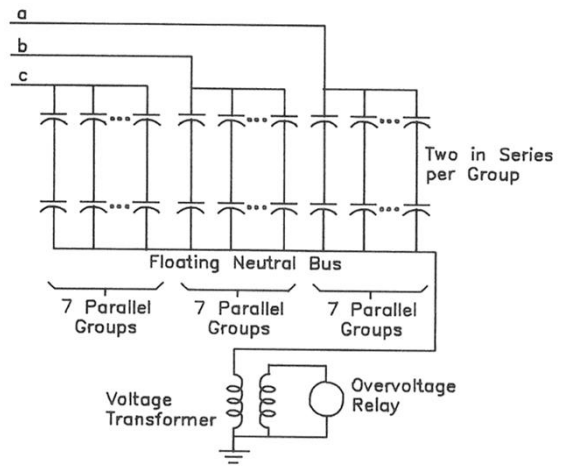


Figure 9-6 Capacitor bank of Example 9-4.

# CONNECTIONS

## FLOATING-WYE CONNECTION

Possible combinations include

- Seven 400-kVAR per phase
- Fourteen 200-kVAR per phase

The voltage rating per phase is equal to the nominal line to neutral voltage of the system.

$$V_{in} = \frac{69,000}{\sqrt{3}} = 39,840 \text{ V}$$

To achieve the required voltage rating, two 19,920-V capacitor units will be connected in series. Therefore, of the possible combinations listed, fourteen 200-kVAR units are the only practical solution: seven parallel combinations of two in series per phase. Each capacitor will have two bushings. The connection is shown in Fig. 9-6.



Because of the higher current rating of these larger banks, individual capacitor unit fusing is recommended, rather than group fusing of the entire bank. To detect individual blown fuses, a voltage transformer may be connected between the floating neutral point of the bank and the substation ground grid. Under normal operating conditions with all capacitor units energized, the voltage between the floating neutral and ground will be essentially zero. In the event of an individual fuse operation, the voltage between the floating neutral and ground will increase. An overvoltage relay connected to the secondary of the voltage transformer can be set to trip in the event of a capacitor unit failure. This overvoltage relay may cause the circuit breaker or switch protecting the bank to trip or may merely indicate an alarm condition.

The operation of individual fuses will also produce an overvoltage on the remaining capacitors in the phase with the blown fuse. Capacitors are designed to operate satisfactorily at up to 110% of rated voltage. Any voltage higher than this will cause breakdown of the dielectric and subsequent failure. The following example illustrates the method used to calculate the neutral to ground voltage and the voltage across the remaining capacitors in a floating-wye bank.

EXAMPLE

A 12.47-kV, 3600-kVAR, floating-wye connected capacitor bank is made up of six 200-kVAR, 7200-V capacitor units per phase. The bank is connected to a 12.47-kV, three-phase, MGN system bus. Assume that a fuse element blows on a single unit connected in phase A. Calculate the voltage across the remaining capacitor units on all three phases and the voltage between the floating neutral point of the capacitor bank and ground.

*Solution* The capacitor units will be represented as constant-impedance elements. The impedance of each unit is

$$Z_{\text{cap}} = -j \frac{7200^2}{200,000} = -j259.2 \Omega$$

In the *B* and *C* phases, there are six parallel units. The combined impedance is

$$Z_B = Z_C = \frac{-j259.2}{6} = -j43.2 \Omega$$

# CONNECTIONS

# FLOATING-WYE CONNECTION

## EXAMPLE (Continued)

In the A phase, there are five parallel units due to the blown fuse. The combined impedance is

$$Z_A = -j51.84 \Omega$$

The source voltages are selected as

$$V_{AN} = 7200\angle 0^\circ \text{ V}, \quad V_{BN} = 7200\angle -120^\circ \text{ V}, \quad V_{CN} = 7200\angle 120^\circ \text{ V}$$

The equivalent circuit model is shown in Fig. 9-7, with mesh currents  $I_1$  and  $I_2$  indicated. Writing loop equations for  $I_1$  and  $I_2$  results in the following:

$$\text{Loop } I_1: I_1(-j43.2) + (I_1 - I_2)(-j51.84) = V_{CN} - V_{AN} = 12,470\angle 150^\circ$$

$$\text{Loop } I_2: (I_2 - I_1)(-j51.84) + I_2(-j43.2) = V_{AN} - V_{BN} = 12,470\angle 30^\circ$$

Gathering terms and arranging in matrix format,

$$\begin{bmatrix} -j95.04 & +j51.84 \\ +j51.84 & -j95.04 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12,470\angle 150^\circ \\ 12,470\angle 30^\circ \end{bmatrix}$$

# CONNECTIONS

# FLOATING-WYE CONNECTION

## EXAMPLE (Continued)

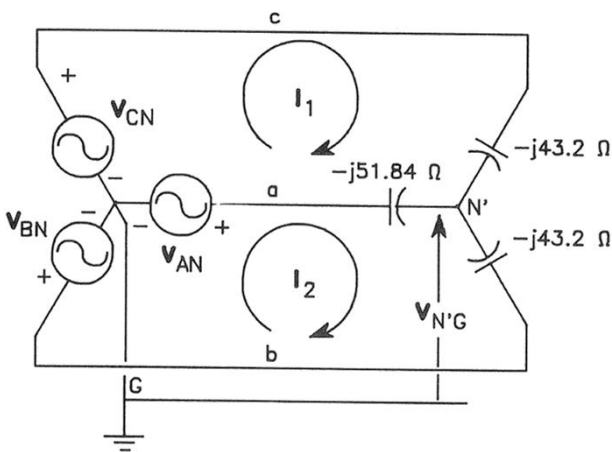


Figure 9-7 Circuit for Example 9-5.

EXAMPLE  
(Continued)

Solving this system of equations for  $I_1$  and  $I_2$  results in the following:

$$I_1 = 162\angle -153^\circ \text{ A}, \quad I_2 = 162\angle -207^\circ \text{ A}$$

From the circuit diagram of Fig. 9-7, the capacitor bank line currents are

$$I_A = I_2 - I_1 = 162\angle -207^\circ - 162\angle -153^\circ = 147.1\angle 90^\circ \text{ A}$$

$$I_B = -I_2 = 162\angle -27^\circ \text{ A}$$

$$I_C = I_1 = 162\angle -153^\circ \text{ A}$$

The voltages across each capacitor phase group are equal to

$$V_{AN'} = I_A \cdot Z_A = (147.1\angle 90^\circ)(51.84\angle -90^\circ) = 7625.7\angle 0^\circ \text{ V}$$

$$V_{BN'} = I_B \cdot Z_B = (162\angle -27^\circ)(43.2\angle -90^\circ) = 6997.5\angle -117^\circ \text{ V}$$

$$V_{CN'} = I_C \cdot Z_C = (162\angle -153^\circ)(43.2\angle -90^\circ) = 6997.5\angle -243^\circ \text{ V}$$

EXAMPLE  
(Continued)

Expressed as a percentage of rated voltages, the voltages across each capacitor phase group are

$$|V_{AN'}| = \frac{7625.7}{7200} \cdot 100 = 106\%$$

$$|V_{BN'}| = |V_{CN'}| = \frac{6997.5}{7200} \cdot 100 = 97.2\%$$

Note that the voltage across the remaining capacitors in phase group  $A$  has increased to 106% of rated voltage, while the voltage in phase groups  $B$  and  $C$  has decreased to 97.2% of rated. These changes are due to the neutral shift that occurs in a floating-wye connected bank.

The voltage between the floating neutral point of the capacitor bank and ground can be obtained by applying Kirchhoff's voltage law around the  $A$  phase loop, resulting in the following equation:

$$\begin{aligned} V_{N'G} &= V_{AN} - I_A \cdot Z_A = V_{AN} - V_{AN'} \\ &= 7200\angle 0^\circ - 7625.7\angle 0^\circ \\ &= 425.7\angle 180^\circ \text{ V} \end{aligned}$$

EXAMPLE  
(Continued)

If a voltage transformer with a 7200–120-V ratio is connected between floating neutral and ground, the voltage on the secondary will be

$$V = 425.7 \cdot \frac{120}{7200} = 7.1 \text{ V}$$

The calculations shown in Example 9-5 are rather cumbersome to perform. A rule of thumb may be applied to limit the overvoltage on the remaining capacitors in the bank to less than 110% of rated voltage. Simply stated, *use either one capacitor unit per phase or at least four units per phase.*

Two groups of capacitors may be connected in the form of the split-wye connection shown in Fig. 9-2d. Series and parallel combinations of capacitor units may be employed in a manner similar to the floating-wye connection to increase voltage and kVAR ratings. The voltage rating of the capacitor units is selected based on the nominal line to neutral system voltage.

To detect the operation of individual fuses, a current transformer is connected between the floating neutrals of the two groups. Under normal conditions, each group of capacitors represents a balanced three-phase load. The resulting neutral current is therefore equal to zero. In the event of a blown fuse on one or more of the units, an unbalanced current will result. This unbalanced current is detected by the overcurrent relay connected to the current transformer secondary. The overcurrent relay may trip a circuit breaker or switch or merely signal an alarm.

EXAMPLE

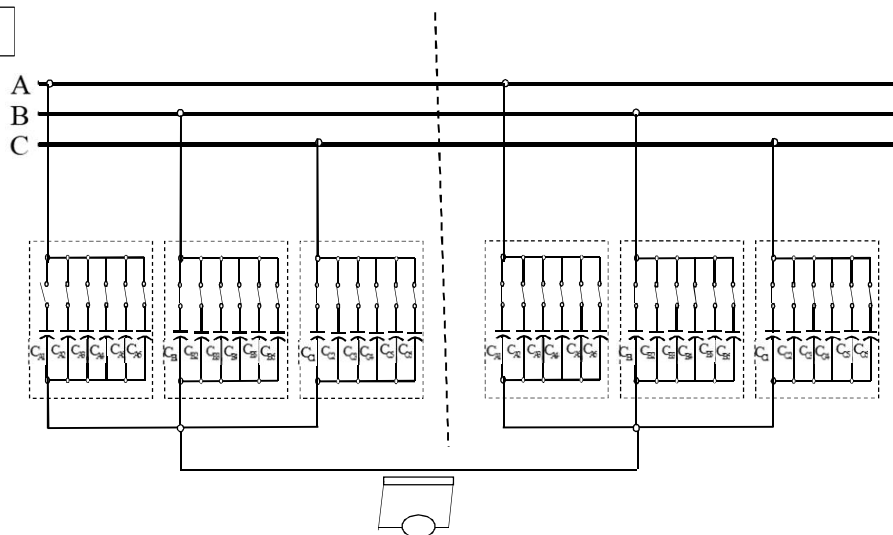
A split-wye capacitor bank is made up of two 3600-kVAR, 12.47-kV sections, for a total installed rating of 7200 kVAR. Each section consists of six 200-kVAR, 7200-V, single-phase capacitor units per phase. Individual fusing is provided on each capacitor unit. A current transformer and overcurrent relay are provided for blown fuse detection. Assume that one capacitor fuse operates in phase A of one of the split-wye sections, and calculate the current flow in the neutral between the two sections.

*Solution* The circuit in Fig. 9-8a models the condition with one fuse blown on one capacitor unit. The first step in determining the neutral current flow between the two sections is to calculate the neutral to ground voltage present on the floating neutral of the bank. The equivalent parallel impedances per phase are calculated with the modified equivalent circuit shown in Fig. 9-8b. The procedure used to calculate the line currents is similar to that shown in Example 9-5.

The loop equations in matrix format are

$$\begin{bmatrix} -j45.2 & +j23.6 \\ +j23.6 & -j45.2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12,470 \angle 150^\circ \\ 12,470 \angle 30^\circ \end{bmatrix}$$

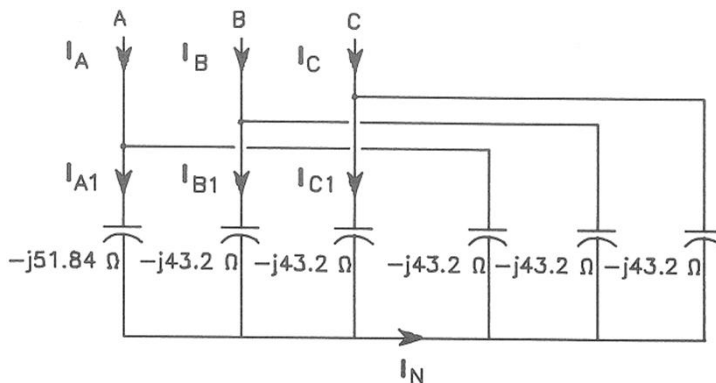
EXAMPLE



# CONNECTIONS

## SPLIT-WYE CONNECTION

### EXAMPLE

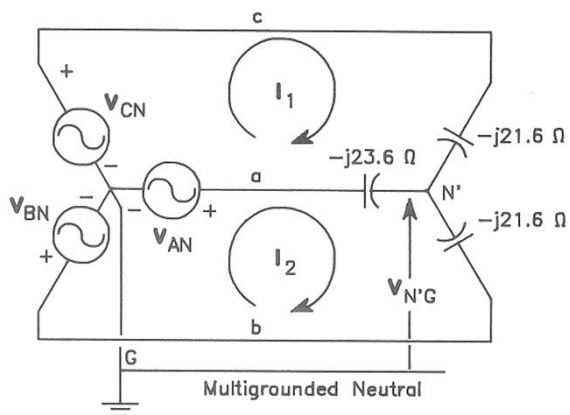


(a) Equivalent Circuit Model.

# CONNECTIONS

## SPLIT-WYE CONNECTION

### EXAMPLE



(b) Reduced Equivalent Circuit.

The loop equations in matrix format are

$$\begin{bmatrix} -j45.2 & +j23.6 \\ +j23.6 & -j45.2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12,470 \angle 150^\circ \\ 12,470 \angle 30^\circ \end{bmatrix}$$

EXAMPLE (Continued)

Solving this system of equations for  $I_1$  and  $I_2$  results in the following:

$$I_1 = 328.6\angle -151.5^\circ \text{ A}, \quad I_2 = 328.6\angle -208.5^\circ \text{ A}$$

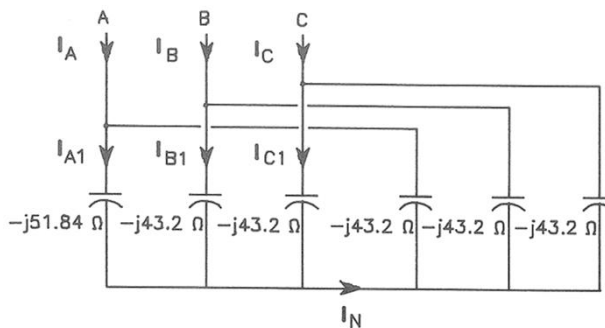
The capacitor bank line currents are

$$I_A = I_2 - I_1 = 328.6\angle -208.5^\circ - 328.6\angle -151.5^\circ = 313.6\angle 90^\circ \text{ A}$$

$$I_B = -I_2 = 328.6\angle -28.5^\circ \text{ A}$$

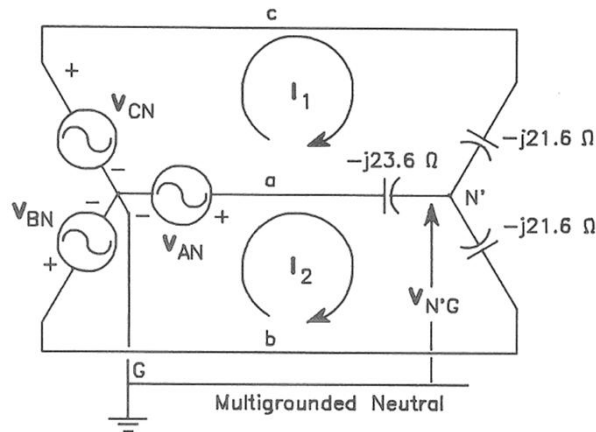
$$I_C = I_1 = 328.6\angle -151.5^\circ \text{ A}$$

EXAMPLE (Continued)



(a) Equivalent Circuit Model.

EXAMPLE (Continued)



(b) Reduced Equivalent Circuit.

EXAMPLE (Continued)

The voltages across each capacitor phase group are equal to

$$V_{AN'} = I_A \cdot Z_A = (313.6 \angle 90^\circ)(23.6 \angle -90^\circ) = 7401 \angle 0^\circ \text{ V}$$

$$V_{BN'} = I_B \cdot Z_B = (328.6 \angle -28.5^\circ)(21.6 \angle -90^\circ) = 7098 \angle -118.5^\circ \text{ V}$$

$$V_{CN'} = I_C \cdot Z_C = (328.6 \angle -151.5^\circ)(21.6 \angle -90^\circ) = 7098 \angle -241.5^\circ \text{ V}$$

Expressed as a percentage of rated voltage, the voltages across each capacitor phase group are

$$|V_{AN'}| = \frac{7401}{7200} \cdot 100 = 102.8\%$$

$$|V_{BN'}| = |V_{CN'}| = \frac{7098}{7200} \cdot 100 = 98.6\%$$

It is interesting to compare the results of this example with Example 9-5. In this example, the voltage across the remaining capacitors in phase group A has increased to 102.8% of rated voltage, while in Example 9-5 the voltage increased to 106% of rated. The difference is that there is less of an imbalance due to the increased number of capacitors per phase in the bank.



EXAMPLE (Continued)

The voltage between the floating neutral point of the capacitor bank and ground can be obtained by applying Kirchhoff's voltage law around the *A* phase loop, resulting in the following equation:

$$\begin{aligned} V_{N'G} &= V_{AN} - I_A \cdot Z_A = V_{AN} - V_{AN'} \\ &= 7200\angle 0^\circ - 7401\angle 0^\circ \\ &= 201\angle 180^\circ \text{ V} \end{aligned}$$

Again, note the comparison between the results of Example 9-5 and this example.

In reference to Fig. 9-8a, the neutral current flowing between the two sections of the split-wye bank is equal to

$$I_N = I_{A1} + I_{B1} + I_{C1}$$

The currents  $I_{A1}$ ,  $I_{B1}$ , and  $I_{C1}$  are given by

EXAMPLE (Continued)

The currents  $I_{A1}$ ,  $I_{B1}$ , and  $I_{C1}$  are given by

$$\begin{aligned} I_{A1} &= \frac{V_{AN'}}{51.84\angle -90^\circ} = \frac{7401\angle 0^\circ}{51.84\angle -90^\circ} = 142.8\angle 90^\circ \text{ A} \\ I_{B1} &= \frac{V_{BN'}}{43.2\angle -90^\circ} = \frac{7098\angle -118.5^\circ}{43.2\angle -90^\circ} = 164.3\angle -28.5^\circ \text{ A} \\ I_{C1} &= \frac{V_{CN'}}{43.2\angle -90^\circ} = \frac{7098\angle -241.5^\circ}{43.2\angle -90^\circ} = 164.3\angle -151.5^\circ \text{ A} \end{aligned}$$

The neutral current flowing between the split-wye connection is

$$\begin{aligned} I_N &= 142.8\angle 90^\circ + 164.3\angle -28.5^\circ + 164.3\angle -151.5^\circ \\ &= 14.0\angle 90^\circ \text{ A} \end{aligned}$$

Therefore, under the specified conditions, approximately 14 A of current will flow between the two sections of the capacitor bank through the current transformer.

## POWER FACTOR IMPROVEMENT

Power capacitors are used to a large extent to improve the power factor of industrial plants and utility power systems. As a result of improved power factor, the apparent power loading and line current are reduced. Reduced line losses are also achieved due to the reduction in line current. Power factor correction capacitors are typically installed in the form of a three-phase bank. Since each phase contains capacitors of equal rating, the result is a balanced three-phase connection.

The ability of capacitors to improve power factor can be understood by examining the one-line diagram for an industrial plant shown in Fig. 9-9. The load is assumed to be operating at a lagging power factor. As such, the load will absorb active power  $P_{LOAD}$  and reactive power  $Q_{LOAD}$  from the system. A capacitor bank added to the bus will supply reactive power  $Q_{CAP}$ . This results in a decrease in the reactive power supplied by the source  $Q_S$  and, consequently, an improved power factor. Unity power factor occurs if the reactive power supplied by the capacitor is equal to the reactive power consumed by the load. Under unity power factor conditions, the source does not supply any reactive power to the load.

## POWER FACTOR IMPROVEMENT

If the reactive power supplied by the capacitor is less than the reactive power consumed by the load, the overall plant power factor will improve, but remain lagging. If the reactive power supplied by the capacitor bank exceeds the reactive power consumed by the load, a leading power factor will result. This condition is sometimes referred to as *overcompensating*. It is generally undesirable to operate a system at a leading power factor.

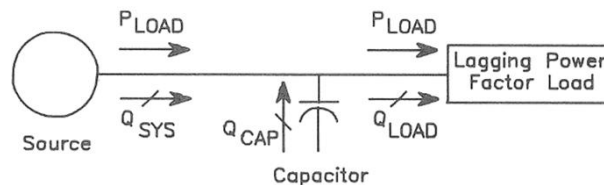


Figure 9-9 One-line diagram illustrating use of capacitor for power factor improvement.

## POWER FACTOR IMPROVEMENT

From Section 1.13, recall the power concepts for single-phase and balanced three-phase loads. In particular, Eqs. (1.67), (1.68), and (1.69) give expressions for the apparent, active, and reactive powers for balanced three-phase loads. These equations are repeated here for reference:

$$S_{3\phi} = \sqrt{3}V_{ll}I_1$$

$$P_{3\phi} = \sqrt{3}V_{ll}I_1 \cos(\theta)$$

$$Q_{3\phi} = -\sqrt{3}V_{ll}I_1 \sin(\theta)$$

The angle  $\theta$  is the power factor angle of the load and is assumed to be positive for leading power factor loads. The quantity  $\cos(\theta)$  is the load power factor (PF), and the term  $\sin(\theta)$  is the reactive factor (RF).

Equations (1.67), (1.68), and (1.69) can be used to express the complex form of the apparent power.

$$S_{3\phi} = P_{3\phi} + jQ_{3\phi}$$

## POWER FACTOR IMPROVEMENT

For actual and desired load conditions, a power triangle can be constructed based on Eq. (9.1) as shown in Fig. 9-10. Here the following variables are defined:

$S_{LOAD}$  = apparent power consumed by the load before adding capacitors

$S_{DES}$  = apparent power supplied by the source after adding capacitors

$Q_{LOAD}$  = reactive power consumed by the load before adding capacitors

$Q_{DES}$  = reactive power supplied by the source after adding capacitors

$P_{LOAD}$  = active power consumed by the load

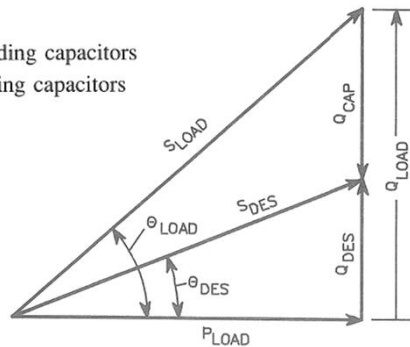
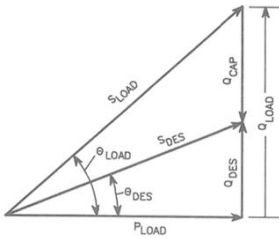


Figure 9-10 Power triangle for system of Fig. 9-9.

## POWER FACTOR IMPROVEMENT



In the analysis that follows, it is assumed that the active power consumed by the load will remain constant both before and after addition of capacitors. The amount of reactive compensation supplied by the capacitor bank is

$$Q_{CAP} = Q_{LOAD} - Q_{DES}$$

The apparent power for the actual and desired power factor conditions can be expressed in terms of the active power demand and actual and desired power factors as follows:

$$S_{LOAD} = \frac{P_{LOAD}}{PF_{LOAD}}$$

$$S_{DES} = \frac{P_{LOAD}}{PF_{DES}}$$

where  $PF_{LOAD}$  and  $PF_{DES}$  are the actual load and desired system power factors, respectively.

The reactive power can be expressed in terms of the apparent power and active power for the actual and desired power factor conditions as follows:

## POWER FACTOR IMPROVEMENT

$$Q_{LOAD} = (S_{ACT}^2 - P_{LOAD}^2)^{1/2}$$

$$Q_{DES} = (S_{DES}^2 - P_{LOAD}^2)^{1/2}$$

Substituting Eq. (9.3) into (9.5) and Eq. (9.4) into (9.6) results in the following:

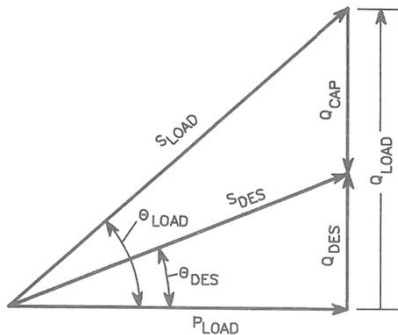
$$Q_{LOAD} = \left[ \left( \frac{P_{LOAD}}{PF_{ACT}} \right)^2 - P_{LOAD}^2 \right]^{1/2}$$

$$Q_{DES} = \left[ \left( \frac{P_{LOAD}}{PF_{DES}} \right)^2 - P_{LOAD}^2 \right]^{1/2}$$

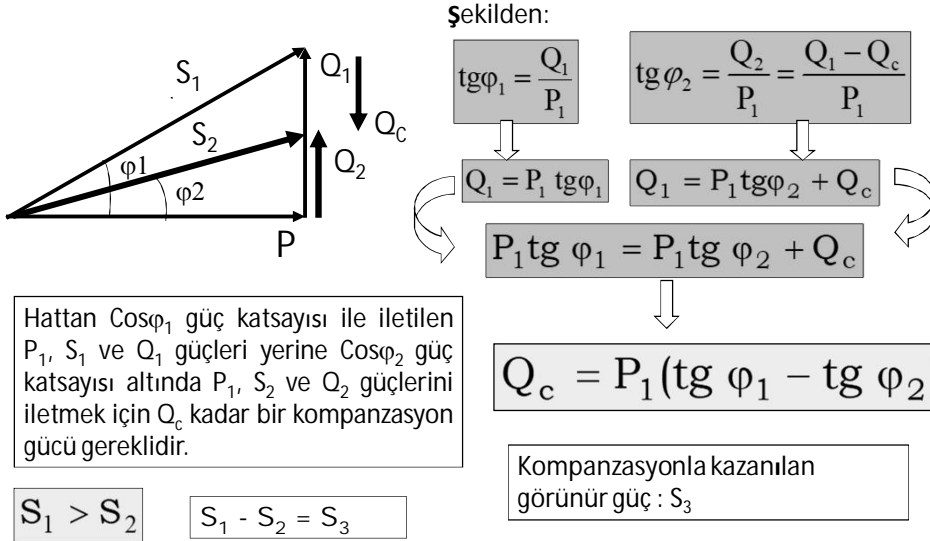
Substituting Eqs. (9.7) and (9.8) into Eq. (9.2) results in

$$\begin{aligned} Q_{CAP} &= \left[ \left( \frac{P_{LOAD}}{PF_{LOAD}} \right)^2 - P_{LOAD}^2 \right]^{1/2} - \left[ \left( \frac{P_{LOAD}}{PF_{DES}} \right)^2 - P_{LOAD}^2 \right]^{1/2} \\ &= P_{LOAD} \cdot \left[ \left( \frac{1}{(PF_{LOAD})^2} - 1 \right)^{1/2} - \left( \frac{1}{(PF_{DES})^2} - 1 \right)^{1/2} \right] \end{aligned}$$

Equation (9.9) can be used to determine the reactive power rating of a capacitor bank required to improve the power factor to a desired level.



## POWER FACTOR IMPROVEMENT



## POWER FACTOR IMPROVEMENT

### EXAMPLE

An industrial plant has an active power demand of 500 kW at a power factor of 0.76 lagging. Determine the required amount of capacitance required to raise the power factor to the following:

- 0.8 lagging
- 0.85 lagging
- 0.9 lagging
- 0.95 lagging
- Unity

Assume that capacitor steps are available in 50-kVAR increments.

*Solution* Direct application of Eq. (9.9) results in the following:

$$\begin{aligned} \text{a. } Q_{\text{CAP}} &= 500 \text{ kW} \cdot \left[ \left( \frac{1}{0.76^2} - 1 \right)^{1/2} - \left( \frac{1}{0.80^2} - 1 \right)^{1/2} \right] \\ &= 52.6 \text{ kVAR} \approx 50 \text{ kVAR} \end{aligned}$$

## POWER FACTOR IMPROVEMENT

### EXAMPLE (Continued)

$$\begin{aligned} \text{b. } Q_{\text{CAP}} &= 500 \text{ kW} \cdot \left[ \left( \frac{1}{0.76^2} - 1 \right)^{1/2} - \left( \frac{1}{0.85^2} - 1 \right)^{1/2} \right] \\ &= 117.7 \text{ kVAR} \approx 100 \text{ kVAR} \end{aligned}$$

$$\begin{aligned} \text{c. } Q_{\text{CAP}} &= 500 \text{ kW} \cdot \left[ \left( \frac{1}{0.76^2} - 1 \right)^{1/2} - \left( \frac{1}{0.90^2} - 1 \right)^{1/2} \right] \\ &= 185.4 \text{ kVAR} \approx 200 \text{ kVAR} \end{aligned}$$

$$\begin{aligned} \text{d. } Q_{\text{CAP}} &= 500 \text{ kW} \cdot \left[ \left( \frac{1}{0.76^2} - 1 \right)^{1/2} - \left( \frac{1}{0.95^2} - 1 \right)^{1/2} \right] \\ &= 263.2 \text{ kVAR} \approx 250 \text{ kVAR} \end{aligned}$$

$$\begin{aligned} \text{e. } Q_{\text{CAP}} &= 500 \text{ kW} \cdot \left[ \left( \frac{1}{0.76^2} - 1 \right)^{1/2} - \left( \frac{1}{1.00^2} - 1 \right)^{1/2} \right] \\ &= 427.6 \text{ kVAR} \approx 400 \text{ kVAR} \end{aligned}$$

Notice that in order to improve the power factor to 0.95 power factor approximately 250 kVAR of capacitors is required. However, to improve the power factor to unity requires 400 kVAR of capacitors.

## POWER FACTOR IMPROVEMENT

### EXAMPLE

A large industrial plant is supplied power at 12.47 kV and has a demand of 4000 kW at a power factor of 0.7 lagging. The source impedance is  $0.5 + j1.3 \Omega/\text{phase}$ . Calculate the amount of capacitors required to improve the power factor to 0.97 lagging. Assume 300-kVAR increments in capacitor size. Also calculate the line current and line  $I^2R$  losses before and after addition of the capacitors.

*Solution* The initial plant load conditions are

$$P_{3\phi} = 4000 \text{ kW}$$

$$S_{3\phi} = \frac{4000 \text{ kW}}{0.7} = 5714.3 \text{ kVAR}$$

$$Q_{3\phi} = (5714.3^2 - 4000^2)^{1/2} = 4080.8 \text{ kVAR}$$

The line current magnitude before adding the capacitors is

$$I_1 = \frac{5714.3 \text{ kVA}}{\sqrt{3} \cdot 12.47 \text{ kV}} = 264.6 \text{ A}$$

## POWER FACTOR IMPROVEMENT

### EXAMPLE (Continued)

The  $I^2R$  losses per phase in the supply line are

$$P_{1\phi} = 264.6^2 \cdot 0.5 = 35,000 \text{ W/phase}$$

For all three phases, the total loss is

$$P_{3\phi} = 3 \cdot P_{1\phi} = 105,000 \text{ W or } 105 \text{ kW}$$

The required capacitor kVAR can be calculated from Eq. (9.9).

$$\begin{aligned} Q_{\text{CAP}} &= 4000 \text{ kW} \cdot \left[ \left( \frac{1}{0.7^2} - 1 \right)^{1/2} - \left( \frac{1}{0.97^2} - 1 \right)^{1/2} \right] \\ &= 3078 \text{ kVAR} \approx 3000 \text{ kVAR} \quad (1000 \text{ kVAR per phase}) \end{aligned}$$

The plant load conditions with the addition of 3000 kVAR of capacitors are

$$P_{3\phi} = 4000 \text{ kW}$$

$$Q_{3\phi} = 4080.8 \text{ kVAR} - 3000 \text{ kVAR} = 1080.8 \text{ kVAR}$$

$$S_{3\phi} = (4000^2 + 1080.8^2)^{1/2} = 4143.2 \text{ kVA}$$

## POWER FACTOR IMPROVEMENT

### EXAMPLE (Continued)

The line current is

$$I_1 = \frac{4143.2 \text{ kVA}}{\sqrt{3} \cdot 12.47 \text{ kV}} = 191.8 \text{ A}$$

The  $I^2R$  losses per phase in the supply line are

$$P_{1\phi} = 191.8^2 \cdot 0.5 = 18,400 \text{ W/phase}$$

For all three phases, the total loss is

$$P_{3\phi} = 3 \cdot P_{1\phi} = 55,200 \text{ W or } 55.2 \text{ kW}$$

Note that the line current has been reduced by 27.5%, while the power loss has been reduced by 47.3 %.

## VOLTAGE IMPROVEMENT

In addition to improved power factor, capacitors will produce a voltage rise on the bus that they are connected to. This voltage rise is due to the leading current of the capacitor being supplied through the inductive reactance of the source. Consider the circuit shown in Fig. 9-11. The rated capacitor current is given by

$$I_C = \frac{Q_{CAP}}{\sqrt{3} \cdot V_{LL}}$$

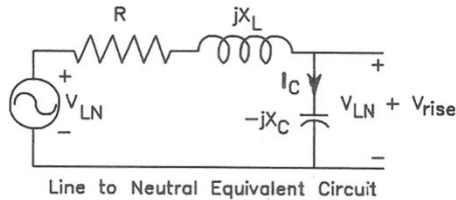


Figure 9-11 Voltage rise due to capacitor.

$Q_{CAP}$  = three-phase reactive power rating of the capacitor in volt-amperes reactive (VAR)

$V_{LL}$  = rated line to line voltage of the capacitor bank in volts

## VOLTAGE IMPROVEMENT

The voltage rise produced by the capacitor is equal to

$$V_{rise} = X_L \cdot I_C$$

where  $X_L$  is the inductive reactance of the line and  $I_C$  is the rated capacitor current. Substituting Eq. (9.10) into (9.11) results in the following:

$$V_{rise} = X_L \cdot \frac{Q_{CAP}}{\sqrt{3} \cdot V_{LL}}$$

The percent voltage rise produced by the capacitor is equal to the actual voltage rise in volts, as given by Eq. (9.12), divided by the nominal line to neutral system voltage, multiplied by 100. In the form of an equation,

$$\%V_{rise} = \frac{V_{rise}}{V_{LN}} \cdot 100$$

Substituting Eq. (9.12) into (9.13) results in

$$\%V_{rise} = X_L \cdot \frac{\left[ \frac{Q_{CAP}}{(\sqrt{3} \cdot V_{LL})} \right]}{V_{LN}} \cdot 100$$



## VOLTAGE IMPROVEMENT

Recall that for balanced three-phase operation

$$V_{LN} = \frac{V_{LL}}{\sqrt{3}}$$

Substituting Eq. (9.15) into (9.14) results in a simplified expression for the voltage rise due to capacitor installation.

$$\%V_{\text{rise}} = X_L \cdot \left[ \frac{Q_{\text{CAP}}}{V_{LL}^2} \right] \cdot 100$$

A more convenient form of Eq. (9.16) is

$$\%V_{\text{rise}} = X_L \cdot \left[ \frac{\text{kVAR}_{\text{CAP}}}{10 \cdot \text{kV}_{LL}^2} \right]$$

## VOLTAGE IMPROVEMENT

### EXAMPLE

A 1200-kVAR, 12.47-kV, three-phase capacitor bank is to be installed at the end of a three-phase distribution feeder 2 miles in length. The impedance of the feeder is  $0.306 + j0.639 \Omega/\text{mile}$ . Calculate the percentage of voltage rise due to this capacitor.

*Solution* The inductive reactance of the line is

$$X_L = (2 \text{ miles}) \cdot (0.639 \Omega/\text{mile}) = 1.278 \Omega$$

Direct application of Eq. (9.17) results in

$$\begin{aligned} \%V_{\text{rise}} &= 1.278 \cdot \left[ \frac{1200}{10 \cdot 12.47^2} \right] \\ &= 0.986\% \approx 1.0\% \end{aligned}$$

Therefore, a voltage rise of approximately 1% will result.

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

In addition to improved power factor and resulting decrease in line current, the installation of capacitors on a distribution circuit will reduce the line losses along the feeder. Example 9-8 illustrated the reduction in  $I^2R$  losses due to the installation of a capacitor bank at the end of a 12.47-kV three-phase feeder. It is desirable to determine the optimal size and location of capacitors to maximize reduction in line losses.

As previously discussed, the capacitor bank will absorb a leading current from the source, while inductive lagging power factor loads absorb a lagging current. The magnitude of the line current can be expressed as follows:

$$I_l = (I_p^2 + I_q^2)^{1/2}$$

$I_p$  = magnitude of in-phase component of line current

$I_q$  = magnitude of quadrature component of line current

The current absorbed by a capacitor bank will subtract from the quadrature component of line current, resulting in the following:

$$I_l = (I_p^2 + (I_q - I_c)^2)^{1/2}$$

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

where  $I_c$  is the magnitude of the capacitor current. For a constant active power load on the circuit, the in-phase component of line current will remain essentially constant. Therefore, the minimum line current magnitude will occur when  $I_c$  is equal to  $I_q$ . When this equality is established, the power factor is unity and the line losses will be minimum.

For concentrated loads, the optimum value of capacitor correction will be the value that corrects the power factor to unity. The optimum capacitor location would be at the load bus itself. Since the load has been corrected to unity power factor, the resulting line current supplied by the source, and hence line losses, will be minimized.

Determining the optimum location and size of capacitor banks to be installed on radial distribution feeders is slightly more complicated than determining the requirements for an industrial plant. Unlike industrial plant buses, the loads on radial distribution feeders are distributed along the length of the circuit or circuit segment. The nature of the distributed loads will vary depending on circuit topography. Large concentrated loads in addition to distributed loads are also very common on distribution circuits.

The time-varying nature of the loads will also be a significant factor in determining capacitor requirements. As discussed in Chapter 4, the loads on a power system change depending on time of day, day of the week, season, and the like. The power factor at which these loads operate will fluctuate as well.

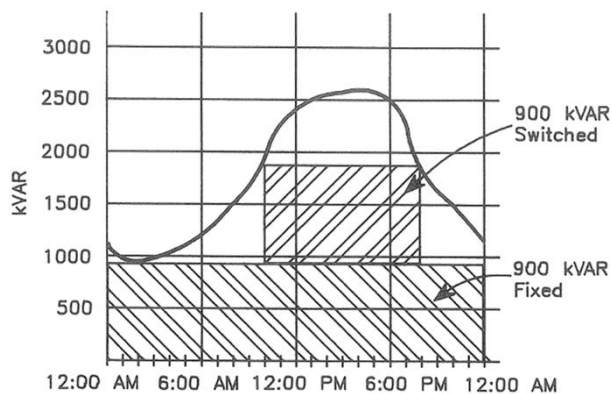
## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

Generally, the rating of fixed capacitor banks installed on a distribution feeder will be equal to the minimum reactive power consumed by the loads connected to the feeder at light load conditions. The voltage rise produced by fixed capacitors must be calculated under light load conditions to ensure that the system voltage is not excessive. Various combinations of switched capacitor units can be applied to supply additional amounts of compensation as the load dictates. Recordings of active and reactive load on the distribution feeders are taken at the distribution substation circuit exits. These recordings are then used to determine the correct amount of fixed and switched capacitors to add to the circuit.

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

### EXAMPLE

Figure 9-12a shows the reactive power load profile characteristic of a 4.16-kV, three-phase, MGN distribution feeder. Determine the amount of fixed and switched capacitor kVAR to be added to correct the circuit power factor.



(a) 900 kVAR Fixed and 900 kVAR Switched.

Figure 9-12 Reactive demand curve, Example 9-10.

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

### EXAMPLE (Continued)

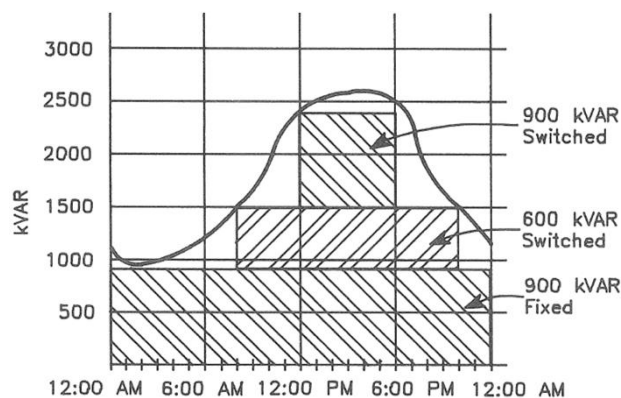
*Solution* From Fig. 9-12a, the minimum reactive power occurs at 2:00 AM and is equal to approximately 970 kVAR. Therefore, a fixed capacitor bank of 900 kVAR (300 kVAR per phase) will be added to the circuit. The switched capacitor requirements will be selected to correct the power factor to as close to unity as possible, without producing a leading power factor. An additional bank of 900 kVAR will be switched on at 10:00 AM and off at 8:00 PM to provide reactive compensation during peak load conditions. Figure 9-12a shows the portion of reactive power supplied by the 900-kVAR fixed bank and the 900-kVAR switched bank.

It is also possible to install two steps of switched capacitor banks to provide a more precise control of reactive power requirements. These two banks may consist of one step of 600 kVAR switched on at 8:00 AM and off at 10:00 PM and a 900-kVAR bank switched on at 12:00 PM and off at 6:00 PM. The addition of additional switched steps is more costly than a single switched step. As always, the increased cost must be

balanced against the benefits provided. Figure 9-12b shows the reactive power supplied by one fixed bank and two switched banks.

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

### EXAMPLE (Continued)



(b) 900 kVAR Fixed, 600 and 900 kVAR Switched.

Figure 9-12 Reactive demand curve, Example 9-10.

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

To determine the most economical size and location of a capacitor bank installed on a radial distribution feeder, consider the circuit segment shown in Fig. 9-13a. The reactive load current is assumed to be uniformly distributed along the length of the line. The reactive current at the beginning of the line segment is designated  $I_1$ . The reactive current at the end of the line is considered to be a lumped load and is designated as  $KI_1$ . Thus, the reactive current at the end of the line segment is expressed as a multiple of the reactive current at the beginning of the line segment. The length of the line is designated as  $l$ . Figure 9-13b shows the reactive load current plotted as a function of distance from the beginning of the line segment.

The equation for the reactive load current as a function of  $x$  is

$$i(x) = \frac{KI_1 - I_1}{l} x + I_1$$

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

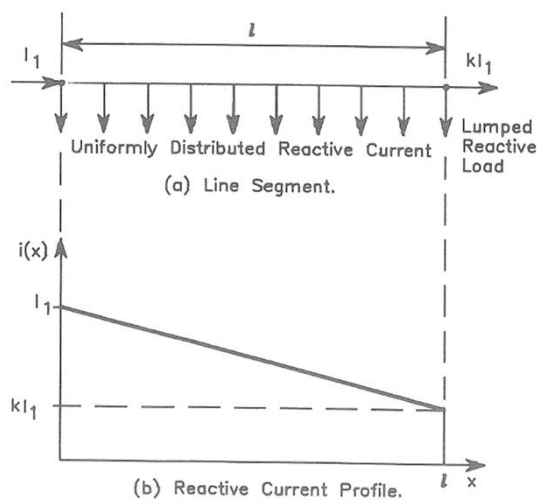


Figure 9-13 Reactive load current distribution.

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

The active power loss per phase due to the reactive component of load current is

$$\begin{aligned}
 P_{\text{loss}} &= \int_0^l \left[ \frac{KI_1 - I_1}{l}x + I_1 \right]^2 R \, dx \\
 &= \frac{l}{3} I_1^2 (K^2 + K + 1)R
 \end{aligned}$$

where  $R$  is the resistance per unit length of the line.

Note that if the load on the circuit consisted of only a lumped sum load with no uniformly distributed load the value of  $K$  would be 1. Applying Eq. (9.20) with  $K = 1$  results in

$$P_{\text{loss}} = I_1^2 l R$$

Equation (9.22) is the familiar  $I^2R$  relationship for power loss.

If the load on the circuit consisted of a uniformly distributed load with no concentrated load at the end, the value of  $K$  would be zero. The power loss per phase due to the reactive component of load current would be

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

$$\begin{aligned}
 P_{\text{loss}} &= \frac{l}{3} I_1^2 (K^2 + K + 1)R \\
 &= \frac{l}{3} I_1^2 R
 \end{aligned}$$

If a single capacitor bank is added to the circuit, the reactive load profile is modified as shown in Fig. 9-14. The reactive current supplied by the capacitor is equal to  $I_c$ . The equation for the reactive component of load current must be split up into two parts, depending on which side of the capacitor bank is of interest. The reactive component of load current is given by

$$i(x) = \frac{KI_1 - I_1}{l}x + I_1 - I_c, \quad \text{for } 0 \leq x \leq x'$$

$$i(x) = \frac{KI_1 - I_1}{l}x + I_1, \quad \text{for } x' \leq x \leq l$$

The power loss per phase with the addition of the capacitor is

LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

$$\begin{aligned}
 P_{\text{loss}} &= \int_0^{x'} \left[ \frac{KI_1 - I_1}{l} x + I_1 - I_c \right]^2 R dx \\
 &+ \int_{x'}^l \left[ \frac{KI_1 - I_1}{l} x + I_1 \right]^2 R dx \\
 &= \left\{ \frac{x'^2}{l} (I_1 I_c (1 - K)) + x' (I_c^2 - 2I_1 I_c) + \frac{l}{3} \left[ I_1^2 (K^2 + K + 1) \right] \right\} \cdot R
 \end{aligned}$$

LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

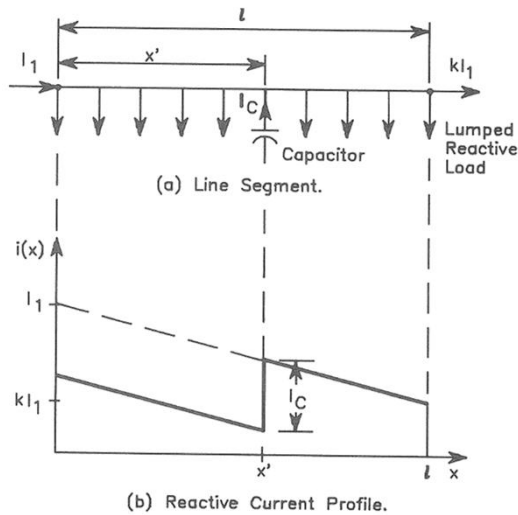


Figure 9-14 Reactive load current distribution with capacitor added.

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

For a given load profile, line length, and resistance, the quantities  $K$ ,  $I_1$ ,  $R$ , and  $l$  are constant in Eq. (9.26). The only two variables are  $I_c$  and  $x'$ , the capacitor size and location, respectively. To determine the optimum capacitor size and location to minimize circuit losses, the partial derivatives of Eq. (9.26) are taken with respect to the two variables  $I_c$  and  $x'$  and equated to zero. The following equations result:

$$\begin{aligned}\frac{\partial P_{\text{loss}}}{\partial x'} = 0 &= \frac{2x'}{l} [I_1 I_c (1 - K)] + (I_c^2 - 2I_1 I_c) \\ \frac{\partial P_{\text{loss}}}{\partial I_c} = 0 &= \frac{x'^2}{l} [I_1 (1 - K)] + 2x' I_c - 2I_1 x' \\ &= \frac{x'}{l} [I_1 (1 - K)] + 2I_c - 2I_1\end{aligned}$$

Equations (9.27) and (9.28) must be solved simultaneously to determine the optimum values of  $I_c$  and  $x'$ . Solving Eq. (9.27) for  $x'$  results in

$$x' = \frac{-(I_c^2 - 2I_1 I_c) l}{I_1 I_c (1 - K) 2}$$

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

It is convenient to express the capacitor current  $I_c$  as a function of the reactive load current  $I_1$  as follows:

$$I_c = \alpha I_1$$

Substituting Eq. (9.30) into (9.29) and simplifying results in

$$x' = \frac{l [2 - \alpha]}{2 [1 - K]}$$

Equation (9.31) gives the optimum location for a given capacitor size and ratio  $K$ .

Substituting Eq. (9.30) into (9.28) and simplifying,

$$\begin{aligned}\frac{\partial P_{\text{loss}}}{\partial I_c} = 0 &= \frac{x'}{l} [I_1 (1 - K)] + 2\alpha I_1 - 2I_1 \\ &= \frac{x'}{l} (1 - K) + 2\alpha - 2\end{aligned}$$



## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

Substituting Eq. (9.31) into (9.32) and simplifying,

$$\begin{aligned} 0 &= \frac{\frac{l}{2} \left[ \frac{(2 - \alpha)}{(1 - K)} \right] (1 - K)}{l} + 2\alpha - 2 \\ &= \frac{2 - \alpha}{2} + 2\alpha - 2 \\ &= \frac{3}{2}\alpha - 1 \end{aligned}$$

Solving Eq. (9.33) for  $\alpha$  results in  $\alpha = 2/3$ .

From the previous derivation, it would appear that the optimum-sized capacitor would have a current rating equal to two-thirds times the reactive load current at the beginning of the line segment. A capacitor rating of two-thirds times the reactive current at the beginning of the line segment does provide optimum line loss reductions up to a  $K$  value of  $1/3$ . This result can be verified by substituting  $\alpha = 2/3$  into Eq. (9.31) and solving for  $x'$  as a function of  $K$ . The result is

$$\begin{aligned} \text{For } K &\leq \frac{1}{3} \\ \alpha &= \frac{2}{3} \quad \Longrightarrow \quad x' = \frac{2}{3}l \left[ \frac{1}{1 - K} \right] \quad X' : \text{Optimum location} \end{aligned}$$

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

Examination of Eq. (9.34) indicates that for values of  $K$  greater than  $1/3$  the value of  $x'$  exceeds  $l$ . This value of  $x'$  is of course longer than the line segment itself. As such, the optimum size of  $2/3$  is only valid for  $K$  less than or equal to  $1/3$ .

If  $K$  exceeds  $1/3$ , the optimum location is at  $x' = l$ , which is at the end of the segment. The optimum value of  $\alpha$  will no longer be  $2/3$ , but can be calculated by substituting  $x' = l$  into Eq. (9.32) and solving for  $\alpha$ , as follows:

$$0 = \frac{l}{l}(1 - K) + 2\alpha - 2$$

$$0 = 1 - K + 2\alpha - 2$$

$$\text{from which } \alpha = \frac{K + 1}{2}$$

$$\text{For } K > \frac{1}{3} \quad \Longrightarrow \quad x' = l$$

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

### EXAMPLE

A section of a 12.47-kV distribution line has a length of 3 miles. The reactive power loading was measured as 2000 kVAR at the distribution substation line exit. The reactive power loading at the end of the 3-mile section was estimated as 600 kVAR. Determine the optimum capacitor rating and location to minimize line loss on this section of feeder.

*Solution* The ratio of reactive power at the end of the line section to the reactive power at the beginning of the line section is

$$K = \frac{600 \text{ kVAR}}{2000 \text{ kVAR}} = 0.3$$

Since the ratio  $K$  is less than  $1/3$ , the optimum capacitor rating is two-thirds times the reactive power loading at the beginning of the line section.

$$\text{kVAR}_{\text{CAP}} = \frac{2}{3} \cdot 2000 \text{ kVAR} = 1333.3 \text{ kVAR}$$

The optimum capacitor location is given by Eq. (9.34).

$$x' = \frac{2}{3} (3 \text{ miles}) \left( \frac{1}{1 - 0.3} \right) = 2.86 \text{ miles}$$

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

### EXAMPLE (Continued)

Therefore, a capacitor bank of approximately 1333.3 kVAR located a distance of approximately 2.86 miles from the substation is required to minimize line losses on this section of feeder. A capacitor bank rating of either 1200 or 1500 kVAR will be appropriate. The bank will be comprised of either four 100-kVAR or five 100-kVAR single-phase units per phase.

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

### EXAMPLE

An 8.32-kV distribution feeder has two major line sections, as shown in Fig. 9-15. Two capacitor banks are to be installed, one on each line section, to minimize losses. Using the estimated reactive power flows shown, determine the optimum rating and location for each line section.

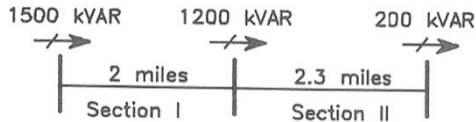


Figure 9-15 Circuit for Example 9-12.

*Solution* The correct procedure is to locate and size the capacitors at the line section farthest away from the source. Next, the reactive profile of the next line section closer to the source will be modified, and the location and size of capacitors for this line section determined. This procedure will result in the minimizing of losses for each line section of interest. Therefore, line section II will be analyzed first, followed by line section I.

SECTION II: The ratio  $K$  is calculated as

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

### EXAMPLE (Continued)

$$K = \frac{200 \text{ kVAR}}{1200 \text{ kVAR}} = 0.167 \quad \implies \quad K < \frac{1}{3}$$

Since  $K$  is less than  $1/3$ , the optimum capacitor bank rating is two-thirds time the reactive load at the beginning of the line section. The optimum capacitor location is determined by applying Eq. (9.34).

$$x' = \frac{2}{3} (2.3 \text{ miles}) \left( \frac{1}{1 - 0.167} \right) = 1.84 \text{ miles} \quad \text{kVAR}_{\text{CAP}} = \frac{2}{3} \cdot 1200 = 800 \text{ kVAR}$$

Therefore, a capacitor bank with a rating of 900 kVAR will be the most practical for this line section. This 900-kVAR bank will be installed approximately 1.84 miles from the beginning of line section II in order to minimize losses in this section of feeder.

SECTION I: With the 900-kVAR capacitor bank installed in line section II, the reactive power flow over line section I needs to be modified. The reactive power flows at the beginning and end of line section I are equal to the initial reactive flows minus the reactive power supplied by the 900-kVAR capacitor bank in line section II. Thus, the reactive power flows at the beginning and end of line section I are equal to 600 and 300 kVAR, respectively.

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

### EXAMPLE (Continued)

The ratio  $K$  is calculated as 
$$K = \frac{300 \text{ kVAR}}{600 \text{ kVAR}} = 0.5$$

Since  $K$  is greater than  $1/3$ , the optimum location of the capacitor bank is at the end of line section I. The optimum capacitor bank rating is determined by applying Eq. (9.35).

$$\alpha = \frac{0.5 + 1}{2} = 0.75$$

The capacitor bank rating is 
$$\text{kVAR}_{\text{CAP}} = 0.75 \cdot 600 = 450 \text{ kVAR}$$

Therefore, a bank rating of 450 kVAR will be installed at the end of line section I to minimize losses on this section of line.

## LOCATION AND SIZING FOR OPTIMAL LINE LOSS REDUCTION

### Summary

If the ratio  $K$  is less than or equal to  $1/3$ , then the optimum-sized capacitor has a rating equal to two-thirds times reactive load at the beginning of the line segment. The optimum location can be calculated by applying Eq. (9.34).

If the ratio  $K$  is greater than  $1/3$ , the optimum capacitor location is at the end of the feeder segment. The optimum capacitor rating is calculated by applying Eq. (9.35).

On most distribution feeders, the two-thirds rule is commonly used to locate and size capacitor banks. This rule states that a capacitor bank rating equal to two-thirds the reactive load at the beginning of a line section should be located a distance of two-thirds the length of the feeder from the source to minimize line losses.

## CAPACITOR SWITCHING

In many instances, it is desirable to install several steps of switched capacitor units, rather than one large fixed bank. This is particularly true if the load reactive power requirements fluctuate by a substantial amount during the day. When a de-energized capacitor is energized, the capacitor essentially behaves as an electrical short. A large current will flow into the capacitor bank under these conditions. The inductance of the source will tend to limit the magnitude of the current flowing into the capacitor bank when energized. Energizing a single bank is referred to as isolated bank switching.

Likewise, when energizing one step of a multistep capacitor bank, a high-magnitude current will result as the energized step discharges into the step being energized. This type of switching is referred to as back to back switching. Generally, back to back switching results in higher-magnitude currents than isolated bank switching.

## CAPACITOR SWITCHING

The calculation of currents during capacitor switching is an extremely important consideration in capacitor applications. Contactors and circuit breakers used for capacitor switching are limited in the amount of momentary current the contacts can safely withstand. This momentary current rating is specified by the switch or breaker manufacturer. Failure to limit momentary current below specified values will result in contact welding, increased pitting of the contacts, or severe damage to the switch.

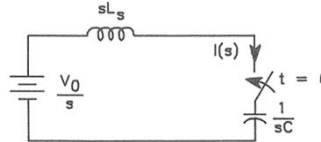
The high-magnitude currents that occur during capacitor switching are also of relatively high frequency compared to the system frequency. These high-frequency currents produce high-frequency voltage spikes on the system that may interfere with the operation of various control circuits. When switching large capacitor banks, it is important to provide overvoltage protection or isolated power supplies for proper operation of the control circuitry.

# CAPACITOR SWITCHING

## Isolated Bank Switching

Exact calculations of capacitor switching currents are extremely difficult to perform manually. To provide some basic guidelines, the following assumptions will be made:

1. The system will be analyzed on a single-phase, line to neutral equivalent basis.
2. The source will be modeled as a dc voltage source.
3. The magnitude of the dc voltage source will be constant.
4. The dc voltage source will have a magnitude equal to the peak line to neutral system voltage.
5. Resistance will be neglected.



The validity of these assumptions lies in the fact that the transient period of interest usually occurs in much less than one cycle of the nominal system frequency. As such, the magnitude of the source voltage will not change substantially during the period of interest.

# CAPACITOR SWITCHING

## Isolated Bank Switching

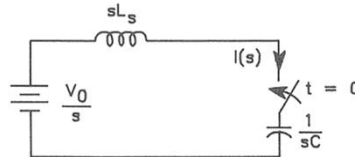
The equivalent circuit based on the preceding assumptions is shown in Fig. 9-16. The magnitude of the source voltage is

$$V_o = \frac{\sqrt{2} \cdot V_{LL}}{\sqrt{3}}$$

An approximate value of source inductance is given by

$$L_s \approx \frac{1}{2 \cdot \pi \cdot f} \cdot \frac{\text{kV}_{LL}^2}{\text{MVA}_{SC}}$$

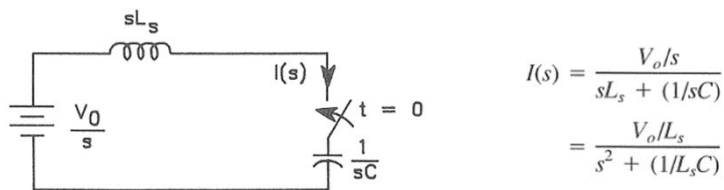
$f$  = system frequency  
 $\text{kV}_{LL}$  = line to line system voltage  
 $\text{MVA}_{SC}$  = three phase short circuit MVA



The capacitance per phase of the capacitor bank is given by

$$C = \frac{\text{MVAR}_{\text{rated}}}{2 \cdot \pi \cdot f_{\text{rated}} \cdot (\text{kV}_{LL,\text{rated}})^2}$$

To determine the capacitor current, the LaPlace transform method is used. Figure 9-16 shows the equivalent circuit for isolated bank switching in the  $s$  domain. In reference to Fig. 9-16, the capacitor current is given by



**Figure 9-16** Equivalent circuit for isolated bank capacitor switching.

Equation (9.39) must be rearranged to a form suitable for taking the inverse LaPlace transform. The following results:

$$I(s) = V_o \left( \frac{C}{L_s} \right)^{1/2} \left( \frac{\omega_o}{s^2 + \omega_o^2} \right) \quad \omega_o = \frac{1}{(L_s C)^{1/2}}$$

$$I(s) = V_o \left( \frac{C}{L_s} \right)^{1/2} \left( \frac{\omega_o}{s^2 + \omega_o^2} \right) \quad \omega_o = \frac{1}{(L_s C)^{1/2}}$$

Equation (9.41) can be used to determine the frequency of the transient inrush current. The inverse LaPlace transform of Eq. (9.40) is

$$i(t) = \mathcal{L}^{-1}I(s) = V_o \left[ \frac{C}{L_s} \right]^{1/2} \sin(\omega_o t)$$

The maximum instantaneous value of the inrush current is

$$I_{\max} = V_o \cdot \left[ \frac{C}{L_s} \right]^{1/2}$$

## EXAMPLE

A 1200-kVAR, 4.16-kV capacitor bank is installed on a plant bus. The plant bus is supplied from a 5000-kVA, 69kV–4.16Y/2.4kV transformer having an impedance of 7%. Neglecting the impedance of the 69kV source and resistance, determine the maximum instantaneous value and the frequency of the inrush current. Also, determine the inductance and current rating of the inductors that must be added to reduce the inrush current to 1000 A.

*Solution* The transformer inductive reactance is  $X = 0.07 \cdot \frac{(4.16 \text{ kV})^2}{5 \text{ MVA}} = 0.2422 \Omega$

The transformer inductance is  $L = \frac{0.2422}{2 \cdot \pi \cdot 60} = 6.425 \times 10^{-4} \text{ H}$

The capacitance per phase is equal to  $C = \frac{1.2 \text{ MVAR}}{2 \cdot \pi \cdot 60 \cdot (4.16 \text{ kV})^2} = 1.839 \times 10^{-4} \text{ F}$

EXAMPLE  
(Continued)

The peak source voltage is equal to  $V_o = \frac{\sqrt{2} \cdot 4160}{\sqrt{3}} = 3396 \text{ V}$

Applying Eq. (9.43) results in  $I_{\max} = 3396 \left( \frac{1.839 \times 10^{-4}}{6.425 \times 10^{-4}} \right)^{1/2} = 1817 \text{ A}$

The frequency of the transient inrush current is

$$\omega_o = \frac{1}{(1.839 \times 10^{-4} \cdot 6.425 \times 10^{-4})^{1/2}} = 2909 \text{ rad/sec}$$

$$= 463 \text{ Hz}$$

To determine the amount of inductance to be added in order to limit the inrush current to 1000 A, Eq. (9.43) is solved for  $L_s$  as follows:

$$L_s = \frac{V_o^2}{I_{\max}^2} C$$



### EXAMPLE (Continued)

Substituting in known values,

$$L_s = \frac{3396^2}{1000^2} \cdot 1.839 \times 10^{-4} = 2.1209 \times 10^{-3} \text{ H}$$

The inductance to be added is equal to

$$L_{\text{ind}} = 2.1209 \times 10^{-3} - 6.425 \times 10^{-4}$$

$$= 1.4784 \times 10^{-3} \text{ H}$$

$$\approx 1.5 \text{ mH}$$

Therefore, an inductor of approximately 1.5 mH should be added in each phase of the capacitor bank in order to limit the inrush current to acceptable values. The inductors used for current limiting are typically air core in design. Iron core inductors are not recommended for current-limiting purposes since the iron will saturate under high current conditions. The result of core saturation is a reduction in the inductance of the inductor. Therefore, the effectiveness of the iron core inductor to limit switching currents will be greatly reduced.

The continuous current rating of the inductors must be equal to the continuous current rating of the capacitor bank.

$$I_{\text{ind}} = \frac{1200 \text{ kVAR}}{\sqrt{3} \cdot 4.16 \text{ kV}} = 166.5 \text{ A}$$

# CAPACITOR SWITCHING

## Back to Back Switching

The equivalent circuit model for back to back capacitor switching is shown in Fig. 9-17. There are a total of  $N$  steps of capacitance that may be switched on and off as reactive power demands dictate. The most severe switching duty will arise when the last step is being energized, with all other steps energized, and the system voltage is at maximum peak value. Under these conditions, the energized steps will discharge into the step being switched on. For the purposes of this analysis, the source contribution will be neglected.

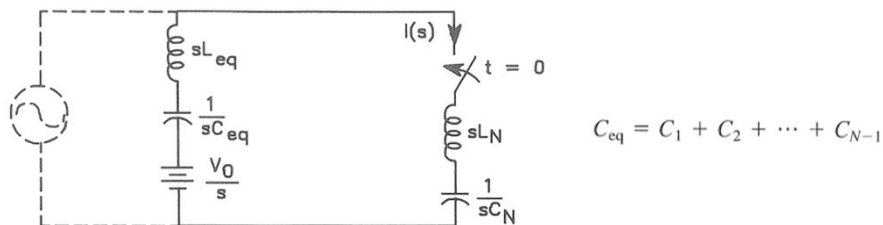


Figure 9-17 Equivalent circuit for back to back capacitor switching.

$$C_{eq} = C_1 + C_2 + \dots + C_{N-1}$$

where  $C_1, C_2, \dots, C_{N-1}$  are the capacitance values per phase of the  $N - 1$  capacitor steps already energized. Each of these capacitance values may be calculated by applying Eq. (9.38) for each step.

Likewise,  $L_{eq}$  represents the equivalent inductance of the current-limiting inductors in series with the  $N - 1$  capacitor steps already energized. This inductance is given by

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_{N-1}}$$

where  $L_1, L_2, \dots, L_{N-1}$  are the values of inductance for each step.

The voltage present on the energized capacitors is assumed to be equal to the peak value of the nominal line to neutral source voltage, as given by Eq. (9.36). In reference to Fig. 9-17, the transient current flowing into the de-energized capacitor bank is

$$I(s) = \frac{V_o/s}{s(L_{eq} + L_N) + (1/sC_{eq}) + (1/sC_N)}$$

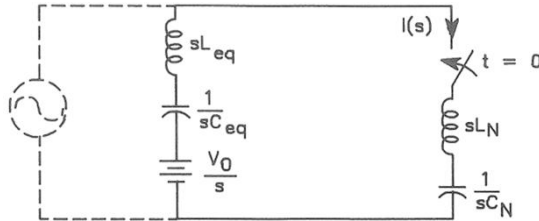
$$\begin{aligned} I(s) &= \frac{V_o/s}{s(L_{eq} + L_N) + (1/sC_{eq}) + (1/sC_N)} \\ &= \frac{V_o/(L_{eq} + L_N)}{s^2 + \frac{1}{C_{eq}(L_{eq} + L_N)} + \frac{1}{C_N(L_{eq} + L_N)}} \\ &= \frac{V_o/(L_{eq} + L_N)}{s^2 + \frac{1/C_{eq} + 1/C_N}{L_{eq} + L_N}} \end{aligned}$$

The natural frequency is

$$\omega_o = \left( \frac{1/C_{eq} + 1/C_N}{L_{eq} + L_N} \right)^{1/2}$$

Substituting Eq. (9.47) into (9.46) and simplifying results in the following:

$$I(s) = V_o \frac{1}{[(L_{eq} + L_N)(1/C_{eq} + 1/C_N)]^{1/2}} \cdot \frac{\omega_o}{s^2 + \omega_o^2}$$



**Figure 9-17** Equivalent circuit for back to back capacitor switching.

The inverse LaPlace transform of Eq. (9.48) is

$$i(t) = \mathcal{L}^{-1}I(s)$$

$$= V_o \frac{1}{[(L_{eq} + L_N)(1/C_{eq} + 1/C_N)]^{1/2}} \sin(\omega_o t)$$

The maximum instantaneous value of the inrush current is

$$I_{max} = V_o \frac{1}{[(L_{eq} + L_N)(1/C_{eq} + 1/C_N)]^{1/2}}$$

Equation (9.49) gives the expression for the transient inrush current flowing into the last step to become energized in a multistep bank. The peak value of the transient inrush current is given by Eq. (9.50) and the frequency by Eq. (9.47).

Equations (9.44) through (9.50) can be simplified when the capacitor bank is made up of equal kVAR-rated steps. The capacitance per phase and the corresponding current-limiting inductors are assumed to have the same value. Let the capacitance and inductance per step be equal to  $C$  and  $L$ , respectively. For  $N - 1$  equal-sized steps, Eqs. (9.44) and (9.45) become

$$C_{eq} = C \cdot (N - 1) \quad L_{eq} = \frac{L}{N - 1}$$

Substituting Eqs. (9.51) and (9.52) into (9.47) and (9.50) results in

$$\omega_o = \frac{1}{(LC)^{1/2}} \quad I_{max} = V_o \cdot \frac{N - 1}{N} \left(\frac{C}{L}\right)^{1/2}$$

To determine the rating of the current-limiting inductors required to limit the maximum peak transient inrush current to a specified value, Eq. (9.54) is solved for the inductance  $L$ , as follows:

$$L = C \cdot \frac{V_o^2}{I_{\max}^2} \cdot \frac{(N - 1)^2}{N^2}$$

### Important Application Note

The derivation of the equations for back to back switching were derived by neglecting the source contribution to switching current. This assumption may not be valid for capacitor banks installed on stiff utility system buses where there is a high availability of fault current. The maximum permissible momentary duty can be derated to 80% of rated to provide a more reasonable estimate of transient switching currents expected. However, if a substantial amount of source contribution is expected, a more detailed analysis may be warranted. This detailed analysis may be performed using a transient analysis package such as the Electromagnetic Transient Analysis Program, or EMTP.

### EXAMPLE

A 13.8-kV, three-step capacitor bank is comprised of three steps of 1800 kVAR each. The vacuum contactor used to switch the steps on and off has a momentary current rating of 10 kA. Neglecting the source contribution and derating the momentary switch rating to 80% of rated, determine the amount of inductance needed to limit the transient inrush current to an acceptable value for the following cases:

- a. One step energized, second step coming on
- b. Two steps energized, third step coming on

Make recommendations as to the proper inductor rating.

*Solution* The capacitance per step is determined by applying Eq. (9.38).

$$C = \frac{1.8 \text{ MVAR}}{2 \cdot \pi \cdot 60 \cdot (13.8 \text{ kV})^2} = 2.5071 \times 10^{-5} \text{ F}$$

The momentary rating of the switch will be derated to

$$I_{\max} = 0.8 \cdot 10,000 \text{ A} = 8000 \text{ A}$$

## EXAMPLE (Continued)

The maximum peak system voltage is

$$V_o = \frac{\sqrt{2} \cdot 13,800}{\sqrt{3}} = 11,266.3 \text{ V}$$

- a. With one step energized and the second step coming on, the number of steps  $N$  is equal to 2. Application of Eq. (9.55) results in the following:

$$\begin{aligned} L &= 2.5071 \times 10^{-5} \cdot \frac{11,266.3^2}{8000^2} \cdot \frac{(2 - 1)^2}{2^2} \\ &= 12.431 \times 10^{-6} \text{ H} \\ &\approx 12.5 \text{ } \mu\text{H} \end{aligned}$$

Therefore, inductors of approximately 12.5 $\mu$ H would be required if this were a two step bank.

EXAMPLE  
(Continued)

- b. With two steps energized and the third step coming on, the number of steps  $N$  is equal to 3. Direct application of Eq. (9.55) results in

$$\begin{aligned} L &= \frac{11,266.3^2}{8000^2} \cdot 2.5071 \times 10^{-5} \cdot \frac{(3 - 1)^2}{3^2} \\ &= 22.1 \times 10^{-6} \text{ H} \\ &\approx 22 \text{ } \mu\text{H} \end{aligned}$$

Therefore, an inductor of approximately 22  $\mu$ H should be installed in each phase of each step in order to limit the transient inrush current to an acceptable level.

The worst-case switching scenario occurs when two steps are energized and the third step is switched on. Comparison of the results of parts a and b indicates that the inductors should have a rating of 22  $\mu$ H. As in Example 9-13, these inductors will be installed in each phase of each step in series with the capacitor units. Since the inductors are series connected, the continuous current rating of the inductors should be equal to the rated current of each capacitor step.

$$I_{\text{ind}} = \frac{1800 \text{ kVAR}}{\sqrt{3} \cdot 13.8 \text{ kV}} = 75.3 \text{ A}$$

## EXAMPLE

A 12.47-kV capacitor bank consists of the following steps:

Step 1: 2400 kVAR

Step 2: 1200 kVAR

Step 3: 600 kVAR

The three steps may be switched at random to provide the required degree of reactive compensation. Determine suitable current-limiting inductor ratings to limit the maximum transient inrush current to 6000 A.

*Solution* Since the capacitor bank is comprised of different kVAR-rated step sizes, Eq. (9.55) is not directly applicable. However, a conservative design approach is to assume that all steps of the bank are equal to the kVAR rating of the largest step and then to apply Eq. (9.55). For this example, the inductor design will be based on three equal steps of 2400 kVAR each. The maximum transient inrush current is then calculated based on *actual* kVAR step sizes and all possible switching combinations. The results of the calculations for all possible switching combinations will be checked to ensure that the maximum transient inrush currents are within design specifications.

EXAMPLE  
(Continued)

The capacitance per phase for the 2400-kVAR step is equal to

$$C_1 = \frac{2.4 \text{ MVAR}}{2 \cdot \pi \cdot 60 \cdot (12.47 \text{ kV})^2} = 40.9 \times 10^{-6} \text{ F}$$

Likewise, for the 1200- and 600-kVAR steps,

$$C_2 = 20.5 \times 10^{-6} \text{ F}, \quad C_3 = 10.2 \times 10^{-6} \text{ F}$$

The peak instantaneous value of the source voltage is

$$V_o = \frac{\sqrt{2} \cdot 12,470}{\sqrt{3}} = 10,180 \text{ V}$$

The number of steps  $N$  is equal to 3. Direct application of Eq. (9.55) results in the following inductance:

$$\begin{aligned} L &= 40.9 \times 10^{-6} \cdot \frac{10,180^2}{6000^2} \cdot \frac{(3-1)^2}{3^2} \\ &= 52.3 \times 10^{-6} \text{ H} \\ &\approx 50 \mu\text{H} \end{aligned}$$

## EXAMPLE (Continued)

Therefore, an inductor having an inductance of approximately 50  $\mu\text{H}$  is required in each step.

With the 2400-kVAR step energized and the 1200-kVAR step being switched on, the following relationships apply.

$$L_{\text{eq}} = L_1, \quad L_N = L_2, \quad C_{\text{eq}} = C_1, \quad C_N = C_2$$

Applying Eq. (9.50) gives the peak transient inrush current.

$$I_{\text{max}} = 10,180 \cdot \frac{1}{[(100 \times 10^{-6})(7.323 \times 10^4)]^{1/2}} = 3762 \text{ A}$$

With the 2400-kVAR step energized and the 600-kVAR step being switched on, the following relationships apply:

$$L_{\text{eq}} = L_1, \quad L_N = L_3, \quad C_{\text{eq}} = C_1, \quad C_N = C_3$$

Applying Eq. (9.50) gives the peak transient inrush current.

## EXAMPLE (Continued)

$$I_{\text{max}} = 10,180 \cdot \frac{1}{[(100 \times 10^{-6})(1.225 \times 10^5)]^{1/2}} = 2909 \text{ A}$$

With the 1200-kVAR step energized and the 600-kVAR step being switched on, the following relationships apply:

$$L_{\text{eq}} = L_2, \quad L_N = L_3, \quad C_{\text{eq}} = C_2, \quad C_N = C_3$$

Applying Eq. (9.50) gives the peak transient inrush current.

$$I_{\text{max}} = 10,180 \cdot \frac{1}{[(100 \times 10^{-6})(1.468 \times 10^5)]^{1/2}} = 2657 \text{ A}$$

With both the 2400- and 1200-kVAR steps energized and the 600-kVAR step being switched on, the following relationships apply:

## EXAMPLE (Continued)

$$L_{eq} = L_1 // L_2 = 25 \times 10^{-6} \text{ H}, \quad L_N = L_3 = 50 \times 10^{-6} \text{ H}$$

$$C_{eq} = C_1 + C_2 = 61.4 \times 10^{-6} \text{ F}, \quad C_N = C_3 = 10.2 \times 10^{-6} \text{ F}$$

Applying Eq. (9.50) gives the peak transient inrush current.

$$I_{max} = 10,180 \cdot \frac{1}{[(75 \times 10^{-6})(1.1433 \times 10^5)]^{1/2}}$$

$$= 3477 \text{ A}$$

The results of this example indicate that the choice of 50- $\mu$ H current-limiting inductors is indeed conservative. This method of selecting current-limiting inductors can provide an initial starting point in the design. The inductor size may be decreased and the transient currents recalculated until a more realistic inductor size is obtained. This is left as an exercise for the student.

## OPERATION AT OFF-RATED VOLTAGE AND FREQUENCY

In some instances, it is necessary to operate capacitors at voltages and frequencies other than rated. The result is a change in the actual reactive power rating of the capacitor bank. The actual reactive power rating of the capacitor bank can be determined for any operating voltage or frequency by recognizing that the capacitance of the capacitor bank is constant regardless of the actual applied voltage or frequency. This value of equivalent line to neutral capacitance can be calculated from Eq. (9.38). The capacitive reactance is given by

$$X_{c,act} = \frac{1}{2 \cdot \pi \cdot f_{act} \cdot C}$$

where  $f_{act}$  is the actual operating frequency of the capacitor, and  $C$  is the capacitance as given by Eq. (9.38). Substituting Eq. (9.38) into (9.56) results in the following:

$$X_{c,act} = \frac{f_{rated}}{f_{act}} \cdot \frac{(\text{kV}_{LL,rated})^2}{\text{MVAR}_{rated}}$$

Equation (9.57) indicates that the capacitive reactance is inversely proportional to the actual operating frequency.



## OPERATION AT OFF-RATED VOLTAGE AND FREQUENCY

The actual line current drawn by the capacitor is

$$I_c = \frac{V_{LN,act}}{X_{c,act}} \\ = \frac{V_{LL,act}/\sqrt{3}}{X_{c,act}}$$

where  $V_{LL,act}$  is the actual line to line operating voltage. Substituting Eq. (9.57) into (9.58) results in

$$I_{c,act} = \frac{f_{act}}{f_{rated}} \cdot \frac{MVAR_{rated}}{(kV_{LL,rated})^2} \cdot \frac{V_{LL,act}}{\sqrt{3}}$$

The actual three-phase reactive power rating of the capacitor bank is

$$MVAR_{act} = \sqrt{3} \cdot (V_{LL,act}) \cdot (I_{c,act}) \times 10^{-6}$$

## OPERATION AT OFF-RATED VOLTAGE AND FREQUENCY

Substituting Eq. (9.59) into (9.60) and simplifying results in

$$MVAR_{act} = MVAR_{rated} \cdot \frac{f_{act}}{f_{rated}} \cdot \frac{(kV_{LL,act})^2}{(kV_{LL,rated})^2}$$

A more convenient form of Eq. (9.61) is

$$kVAR_{act} = kVAR_{rated} \cdot \frac{f_{act}}{f_{rated}} \cdot \frac{(kV_{LL,act})^2}{(kV_{LL,rated})^2}$$

Equation (9.61) or (9.62) can be used to calculate the actual reactive power output of the capacitor bank at off-nominal voltage and frequency. Keep in mind, however, that capacitors should not be operated at above rated voltage.

## OPERATION AT OFF-RATED VOLTAGE AND FREQUENCY

### EXAMPLE

A three-phase capacitor bank is made up of six 200-kVAR, 2770-V, 60-Hz, single-phase capacitor units per phase. The capacitors are floating wye connected and will be applied on a 4.16-kV, 60-Hz plant bus. Determine the actual reactive power rating of the capacitor bank under these conditions.

*Solution* The rated line to line voltage of the capacitor bank is

$$kV_{LL, \text{rated}} = 2770 \cdot \sqrt{3} = 4800 \text{ V}$$

The rated reactive power of the capacitor bank is

$$k\text{VAR}_{\text{rated}} = (6 \text{ units/phase})(200 \text{ kVAR/unit})(3 \text{ phases}) = 3600 \text{ kVAR}$$

The actual reactive power of the bank is determined by applying Eq. (9.62).

$$k\text{VAR}_{\text{act}} = 3600 \text{ kVAR} \cdot \frac{60 \text{ Hz}}{60 \text{ Hz}} \cdot \frac{4160^2}{4800^2} = 2700 \text{ kVAR}$$

Therefore, the reactive power output has been reduced to 75% of nameplate value due to operation at reduced voltage.

## CONTROLS

As previously discussed, capacitor banks may be permanently connected (fixed) or switched as reactive power demands dictate. Fixed capacitors are generally sized to supply reactive power equal to the minimum reactive power demands of the system. Switched capacitors are used to supply the variable amount of reactive power needed as the load increases. Switching of the capacitor banks may be done either manually or automatically. Several types of automatic control can be used to initiate switching of capacitor banks. These include time clock, power factor, and voltage. The type of control selected depends on the primary function of the capacitor bank itself.

Time clocks can be used where the reactive power demand is a predictable function of time. Examination of the reactive load profile for a given distribution feeder or substation bus will indicate the desired switching times. The reactive power consumption of industrial plant loads may also be a predictable function of time based on manufacturing and shift schedules. It must be understood that capacitor switching based on time ignores the actual load conditions and may produce undesirable results. However, capacitor switching based on time of day is the most economical of all automatic switching schemes.

# CONTROLS

If the primary function of the capacitor bank is to control the load power factor to a specified value, a controller that senses the load power factor is the most desirable. This type of control operates independently of time and generally produces the most desirable effect. Typically, a set of current and/or voltage transformers may be needed to provide the appropriate electrical quantities to the controller. The costs associated with controllers based on load power factor are generally higher than time-based control. This is due to the increased complexity of the controller itself, as well as the need for voltage and current transformers.

Control based on voltage is used where the primary function of the capacitor bank is to provide voltage improvement. The capacitor is switched on and off based on the magnitude of the system voltage. An increase in the reactive power demand of a lagging power factor load causes a reduction in the bus voltage. This decrease in voltage is sensed by the controller and switching action is initiated. Conversely, as the reactive power demand of the load decreases, the bus voltage increases. Under these conditions, the controller switches the capacitor steps off to prevent excessive bus voltages from occurring.

# CONTROLS

Switching of multistep capacitor banks is usually performed one step at a time. A time delay between step switching is often included to allow the system to adjust to the new conditions. The amount of time delay is often adjustable by the user. Also, an adjustable reactive power bandwidth is provided to prevent excessive hunting of the controller, thereby minimizing switching operations. Excessive switching will cause premature wear of the switch contacts and control mechanism.

## PROBLEMS

- 9.1. Determine the required voltage and kVAR ratings of the capacitor units used to construct a 2100-kVAR, grounded-wye connected capacitor bank. The bank is to be installed on an 8.32-kV MGN distribution substation bus. Use equal kVAR ratings for all capacitors in the bank. Specify the required number of high-voltage bushings.
- 9.2. The capacitor units in the bank of Problem 9.1 are protected by individual fuse elements. Determine the resulting neutral current if one of the fuse elements blows.
- 9.3. Determine the required voltage and kVAR ratings of the capacitor units used to construct a 1800-kVAR, delta-connected capacitor bank. The bank is to be installed on a 13.8-kV plant bus. Use equal kVAR ratings for all capacitors in the bank. Specify the required number of high-voltage bushings.
- 9.4. Determine the required voltage and kVAR ratings of the capacitor units used to construct a 2700-kVAR, floating-wye connected capacitor bank. The bank is to be installed on a 24.94-kV MGN distribution substation bus. Use equal kVAR ratings for all capacitors in the bank. Specify the required number of high-voltage bushings.
- 9.5. The capacitor units in the bank of Problem 9.4 are protected by individual fuse elements. Determine the resulting neutral to ground voltage and the voltage across the remaining capacitor elements in the bank if one of the fuse elements blows.

## PROBLEMS

- 9.6. A floating-wye capacitor bank is constructed of three 200-kVAR, 2400-V capacitor units per phase. The capacitor units are individually fused. Determine the voltages on the remaining capacitor units in the bank and the neutral to ground voltage if one of the fuse elements blows. Is this an acceptable design? If not, recommend an alternative design. Specify quantity, kVAR, and voltage ratings of capacitors used in the new design.
- 9.7. A split-wye capacitor bank is made up of two 4500-kVAR, 13.8-kV sections, for a total installed rating of 9000 kVAR. Each section consists of five 300-kVAR, 7960-V, single-phase capacitor units per phase. The capacitor units are individually fused. Determine the current flow in the neutral between the two sections if one fuse element blows in one of the sections.
- 9.8. An industrial plant is supplied service at 4.16 kV. The plant has an active power demand of 1200 kW at a power factor of 0.72 lagging. Determine the following:
  - a. Apparent power demand
  - b. Reactive power demand
  - c. Line current magnitude
  - d. Amount of capacitors required to improve the plant power factor to 0.95 lagging
  - e. Active, reactive, and apparent power demands after addition of the capacitors
  - f. Line current magnitude after addition of the capacitors

## PROBLEMS

- 9.9. A 12.47-kV, three-phase, MGN, distribution feeder has a length of 3-miles. The impedance is  $0.129 + j0.591 \Omega/\text{mile}$ . An 1800-kVAR capacitor bank is installed at the end of the feeder. Calculate the percentage of voltage rise due to the installation of this capacitor bank.
- 9.10. A 4-mile section of 12.47-kV, three-phase distribution feeder has a uniformly distributed load and a concentrated load at the end of the line section. The reactive power loading at the beginning of the line section was measured as 3200 kVAR. The reactive power loading at the end of the line section is estimated to be 600 kVAR. Determine the rating and location of a capacitor bank required to minimize line losses along this section of line.
- 9.11. A 3600-kVAR, 13.8-kV capacitor bank is installed on a plant bus supplied from a 7500-kVA, 138 kV–13.8 kV transformer having a nameplate impedance of 6.8%. Neglecting the impedance of the 138-kV source and transformer resistance, determine the maximum instantaneous value and frequency of the transient inrush current when the capacitor bank is switched on.

## PROBLEMS

- 9.12. For the plant in Problem 9.11, it is desired to limit the maximum peak transient inrush current to 1000 A in order to minimize voltage dips during switching. If current-limiting inductors are required, specify the inductance and continuous current rating.
- 9.13. A four-step, 4.16-kV capacitor bank consists of three steps of 1200 kVAR and one step of 600 kVAR. The maximum momentary rating of the switching device is 5000 A. Determine the inductance and continuous current rating of the current-limiting inductors required for this situation.
- 9.14. A 2400-kVAR, 60-Hz, floating-wye connected capacitor bank is constructed of four 200-kVAR, 7960-V, single-phase capacitor units per phase. Calculate the actual reactive power rating if connected to a 12.47-kV bus.

CHAPTER 6  
CAPACITORS IN DISTEIBUTION SYSTEMS

END OF THE CHAPTER